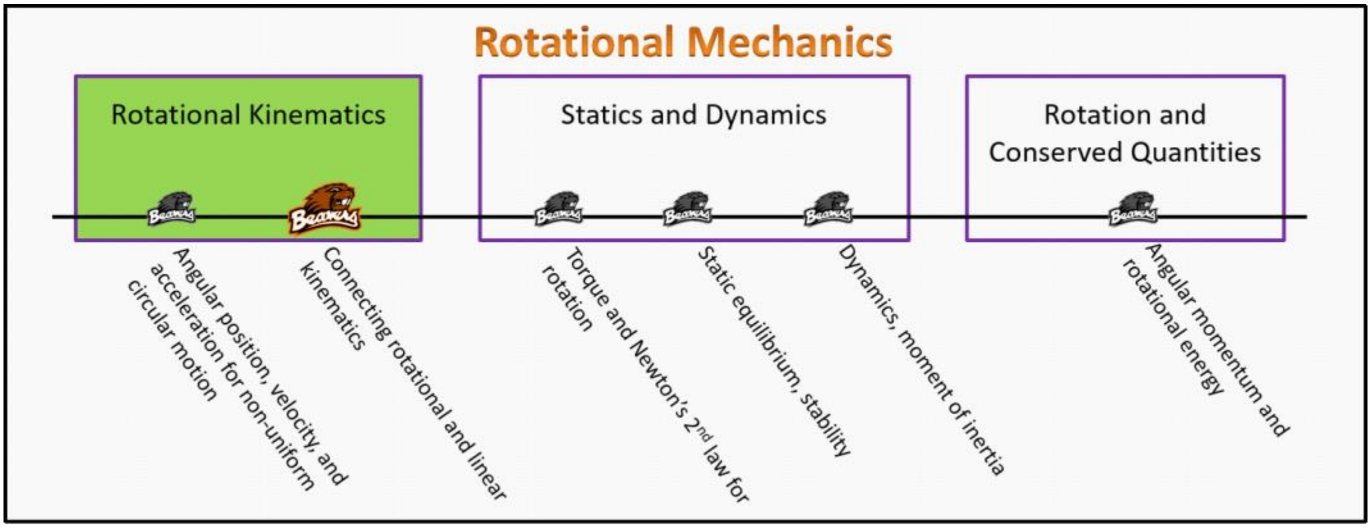
Solutions

Wednesday, January 9, 2019 7:15 AM

Rotational Kinematics
Foundation Stage (RK.2.L2)

lecture 2
Connecting rotational and linear kinematics



Textbook Chapters (* Calculus version)

- **BoxSand** :: KC videos (rotational kinematics)
- **Knight** (College Physics : A strategic approach 3rd) :: 7.1 ; 7.2
- ***Knight** (Physics for Scientists and Engineers 4th) :: 4.4 ; 4.5 ; 4.6 ; 12.1
- **Giancoli** (Physics Principles with Applications 7th) :: 8-2

Warm up

RK.2.L2-1:

Description: Given initial and final angular velocity and time, determine the average angular acceleration.

Learning Objectives: [?] - Can you identify the objectives from the previous lecture, and this lecture, that this question is relevant to?

Problem Statement: It takes about 0.75 seconds to start a car. When the car is started, the engine's crankshaft is spinning at 1,500 RPM. What is the average acceleration of the engine's crankshaft?

(1) 33.3 rad/s^2
(2) 118 rad/s^2
(3) 209 rad/s^2
(4) 2000 rad/s^2

$$\bar{\alpha} = \frac{\Delta \omega}{\Delta t}$$

$$\bar{\alpha} = \frac{\omega_f - \cancel{\omega_i}^{\,0}}{\Delta t}$$

$$\bar{\alpha} = \frac{\omega_f}{\Delta t}$$

$$\bar{\alpha} = \frac{2\pi f_f}{\Delta t}$$

$$\bar{\alpha} = \frac{2\pi\left(25 \frac{REV}{S}\right)}{0.75 \text{ SEC}}$$

$$\boxed{\bar{\alpha} \simeq 209 \frac{RAD}{S^2}}$$

SI

$$1500 \frac{REV}{MIN} \times \frac{1 \text{ MIN}}{60 \text{ SEC}} = 25 \frac{REV}{SEC}$$

Selected Learning Objectives

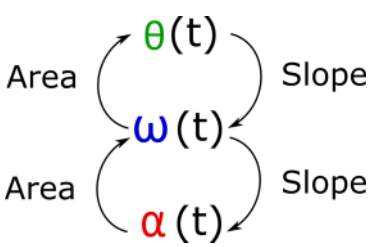
1. Coming soon to a lecture template near you.

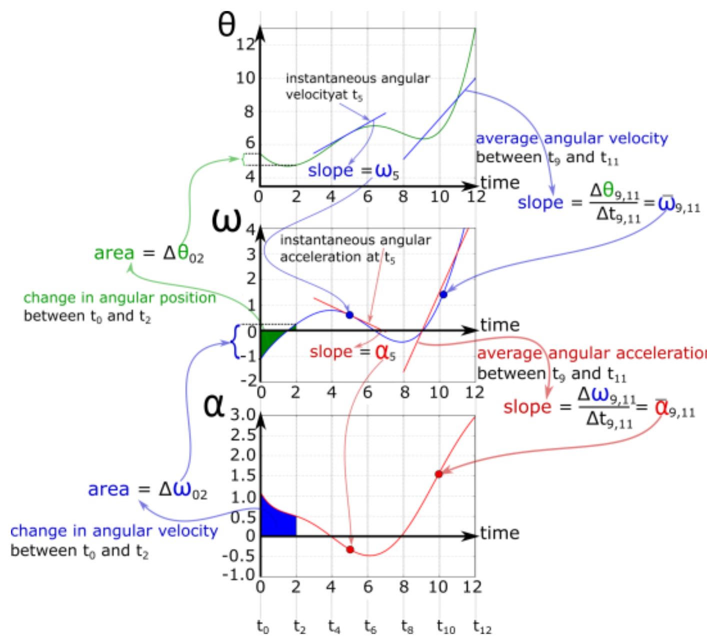
Key Terms

- Arc length (i.e. linear distance or distance)
- Rotational kinematics physical representation

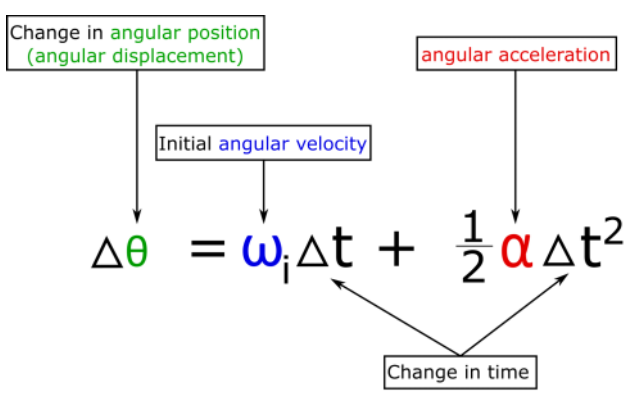
Key Equations

GRAPHICAL ANALYSIS





ROTATIONAL KINEMATICS



In words: The change in the angular position is equal to the initial angular velocity multiplied by the change in time plus one-half of the angular acceleration multiplied by the change in time squared.

$$\overbrace{\omega_f}^{\text{Final angular velocity}} = \overbrace{\omega_i}^{\text{Initial angular velocity}} + \overbrace{\alpha}^{\text{angular acceleration}} \overbrace{\Delta t}^{\text{Change in time}}$$

In words: The final angular velocity is equal to the initial angular velocity plus the angular acceleration multiplied by the change in time.

$$\overbrace{\omega_f^2}^{\text{Final angular velocity}} = \overbrace{\omega_i^2}^{\text{Initial angular velocity}} + 2\overbrace{\alpha}^{\text{angular acceleration}} \overbrace{\Delta\theta}^{\text{Change in angular position (angular displacement)}}$$

In words: The final angular velocity squared is equal to the initial angular velocity squared plus two times the angular acceleration multiplied by the change in the angular position.

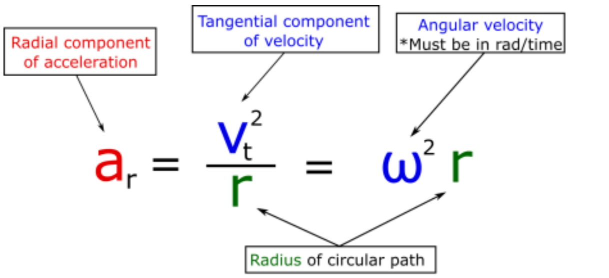
CONNECTING ROTATIONAL TO LINEAR

$$\overbrace{s}^{\text{Arc length}} = \underbrace{r}_{\text{Radius of circular path}} \overbrace{\Delta\theta}^{\text{Change in angular position (angular displacement) *Must be in radians}}$$

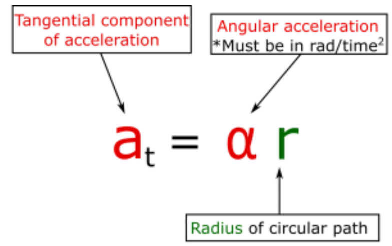
$$\overbrace{v_t}^{\text{Tangential component of velocity}} = \overbrace{\omega}^{\text{Angular velocity *Must be in rad/time}} \underbrace{r}_{\text{Radius of circular path}}$$

In words: The arc length (i.e. linear distance) an object travels is equal to the radius of the circular path the object is traveling multipled by the angular displacement the object goes through.

In words: The tangential component of velocity is equal to the angular velocity times the radius of the circular path the object is traveling around.



In words: The radial component of acceleration is equal to the tangential component of velocity squared divided by the radius of the circular path the object is traveling. The radial component of acceleration is also equal to the angular velocity squared times the radius of the circular path the object is traveling.



In words: The tangential component of acceleration is equal to the angular acceleration times the radius of the circular path the object is traveling.

Key Concepts

- Graphical analysis for rotational kinematics is analogous to graphical representation for regular linear kinematics.
- A physical representation for a rotational kinematics analysis should include the following quantities: trajectory (for rotational kinematics this will always be a circle or circular arc), at least 2 dots representing the object at two different snapshots in time, angular velocity at each snapshot represented with curvy lines to show weather positive or negative, angular acceleration between a set of snapshots represented with a curvy line to show weather positive or negative, and the angular displacement between a set of snapshots represented with a curvy line to show weather positive or negative.
- The arc length (S) is a positive scalar quantity that represents the linear distance an object would have traveled if it the object traveled in a straight line rather than in a circle. In general, the arc length is the distance along a curved path. When using the mathematical representation of arc length, be sure to use units of radians for the angular displacement else the above key equation won't result in the proper arc length.
- Linear quantities such as distance traveled can be related to rotational quantities.

Questions

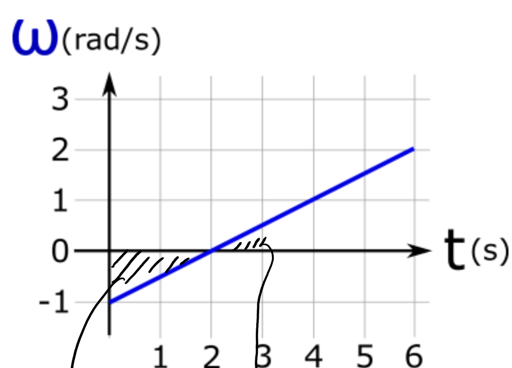
Act I: Graphical representation - slopes and areas

RK.2.L2-2:

Description: Use an angular velocity vs time graph to determine angular acceleration and angular displacement. Sketch the corresponding angular acceleration and angular displacement graphs as a function of time given angular velocity vs time. (1 minute + 2 minutes + 3 minutes + 2 minutes + 3 minutes)

Learning Objectives: [1, 12, 13]

Problem Statement: Danny Johnson the DJ spins a disk whose angular velocity is plotted as a function of time as shown below. The disk initially started at **θ** = 0 radians.



(a) What is the angular velocity of the disk at **t** = 3 seconds?

$\omega(t=3s) \approx \boxed{.5 \frac{RAD}{s}}$

(b) What is the angular acceleration of the disk at **t** = 3 seconds?

$\left.\begin{matrix}\omega(t)\\ \alpha(t)\end{matrix}\right\}$ SLOPE

$\alpha = \frac{(1-0)\frac{RAD}{s}}{(4-2)s} = \boxed{0.5 \frac{RAD}{s^2}}$

(c) What is the angular displacement of the disk from **t** = 0 to **t** = 3 seconds?

AREA $\left\{\begin{matrix}\theta(t)\\ \omega(t)\end{matrix}\right.$

$AREA = \Delta\theta = \frac{1}{2}(2s)(-1 \frac{RAD}{s}) + \frac{1}{2}(1s)(0.5 \frac{RAD}{s})$

AREA $\{ \omega(+)$

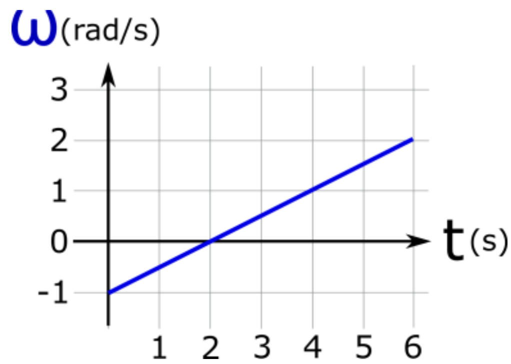
$$\text{AREA} = \Delta\theta = \tfrac{1}{2}(2\,s)(-1\,\tfrac{\text{RAD}}{s}) + \tfrac{1}{2}(1\,s)(0.5\,\tfrac{\text{RAD}}{s})$$

$$= -1\,\text{RAD} + 0.25\,\text{RAD}$$

$$\boxed{\Delta\theta = -0.75\,\text{RAD}}$$

(d) During which of the following time intervals is the disk slowing down?

(1) From **t** = 0 to **t** = 2 seconds.
(2) From **t** = 2 to **t** = 6 seconds. $\longrightarrow$ SPEEDING UP
(3) From **t** = 0 to **t** = 6 seconds.
(4) The slope is positive so the disk is never slowing down.



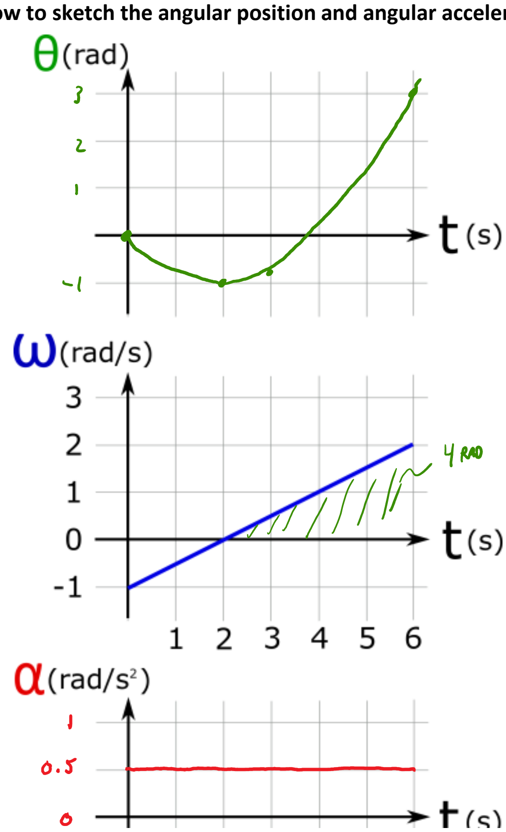
LOOK FOR $|\omega| \downarrow$

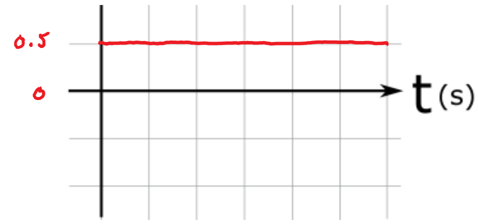
.... OR...

$\omega(-)$ + $\alpha(+)$

$\omega(+)$ + $\alpha(-)$

(e) Danny Johnson the DJ spins a disk whose angular velocity is plotted as a function of time as shown below. The disk initially started at **θ** = 0 radians. **Use the provided axes below to sketch the angular position and angular acceleration of the disk as functions of time.**





RK.2.L2-3:

Description: Given angular position vs time graph determine characteristics of angular velocity and acceleration at a given instant of time. (4 minutes)

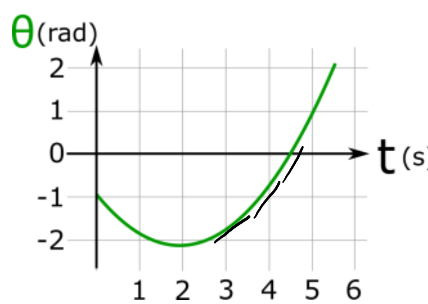
Learning Objectives: [1, 12, 13]

Problem Statement: Three students are discussing the graph shown below. Which of the following do you agree with the most?

(1) At **t** = 3 seconds, the object is in the positive angular position region, moving towards the negative angular position region, and slowing down.

(2) At **t** = 3 seconds, the object is in the negative angular position region, moving towards the positive angular position region, and slowing down.

(3) At **t** = 3 seconds, the object is in the negative angular position region, moving towards the positive angular position region, and speeding up.



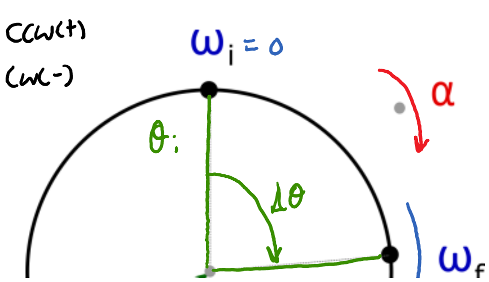
Act II: Rotational kinematics

RK.2.L2-4:

Description: Rotational kinematics problem solving for angular displacement. (6 minutes)

Learning Objectives: [1, 12, 13]

Problem Statement: A CD starts from rest and accelerates at a constant rate of 12.6 rad/s² until it reaches 31.5 rad/s. Through how many radians does the CD travel through from rest to 31.5 rad/s?



$\Delta\theta$ ω_i ω_f α Δt	
K	UK
$\omega_i = 0$ $\omega_f = -31.5$ rad	$\Delta\theta$

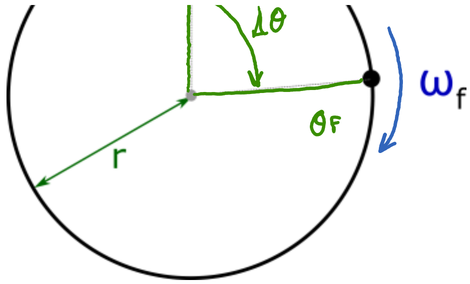
$$\Delta\theta = \omega_i \,\Delta t + \tfrac{1}{2}\,\alpha\,\Delta t^2$$

$$\omega_f = \omega_i + \alpha\,\Delta t$$

2 Eqn
2 unkns

$$\omega_f^2 = \omega_i^2 + 2\,\alpha\,\Delta\theta$$

1 Eqn
1 unkn



K	UK
$\omega_i = 0$	$\Delta\theta$
$\omega_f = -31.5\ \frac{RAD}{S}$	Δt
$\alpha = -12.6\ \frac{RAD}{S^2}$	

$$\omega_f^2 = \omega_i^2 + 2\,\alpha\,\Delta\theta$$

1 Eqn
1 Unkn

$$\omega_f^2 = \cancel{\omega_i^2}^{\,0} + 2\alpha\Delta\theta$$

$$\omega_f^2 = 2\alpha\Delta\theta$$

$$\Delta\theta = \frac{\omega_f^2}{2\alpha} = \frac{\left(-31.5\ \frac{RAD}{S}\right)^2}{2\left(-12.6\ \frac{RAD}{S^2}\right)} = \boxed{-39.4\ RAD}$$

(1) 0 rad.

(2) -1.25 rad.

(3) 2.5 rad.

(4) -39.4 rad.

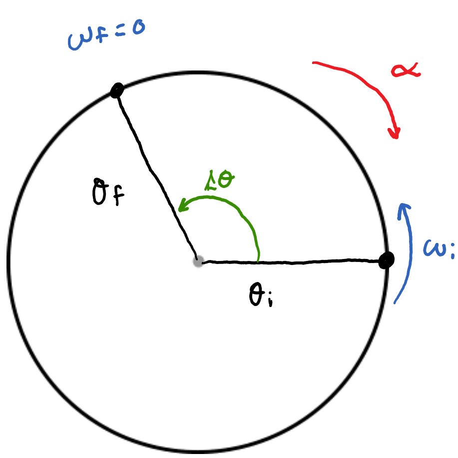
(5) 397 rad

RK.2.L2-5:

Description: Rotational kinematics problem solving for angular displacement. (8 minutes)

Learning Objectives: [1, 12, 13]

Problem Statement: Boeing 737 planes typically use some variant of a CFM56 jet engine. An aircraft mechanic records the time, 45 seconds, it take the engine's core to come to rest after it's shut down. A sensor on the axel of the engine core is used to find the magnitude of average angular acceleration which was about 19.8 rad/s^2. Through how many radians did the jet engine core go through from idle speed to rest?



CCW(+)
CW(-)

Δθ	ω_i	ω_f	α	Δt

K	UK
$\omega_f = 0$	$\Delta\theta$
$\alpha = -19.8\ \frac{RAD}{S^2}$	ω_i
$\Delta t = 45\ SEC$	

$$\Delta\theta = \omega_i\,\Delta t + \tfrac{1}{2}\,\alpha\,\Delta t^2$$

$$\omega_f = \omega_i + \alpha\,\Delta t$$

2 Eqn
2 Unkn's

$$\omega_f^2 = \omega_i^2 + 2\,\alpha\,\Delta\theta$$

2 Eqns
2 Unkn

(1) 891 rad.

(2) -19200 rad.

(3) 39600 rad.

(4) 20048 rad.

$\cancel{\omega_f}^{0} = \omega_i + \alpha \Delta t$

$0 = \omega_i + \alpha \Delta t$

$\omega_i = -\alpha \Delta t$

$= -(-19.8 \frac{\text{RAD}}{s^2})(45\,s)$

$\omega_i = 891 \frac{\text{RAD}}{s}$

$\Delta\theta = \omega_i \Delta t + \frac{1}{2}\alpha \Delta t^2$

$= (891 \frac{\text{RAD}}{s})(45\,s) + \frac{1}{2}(-19.8 \frac{\text{RAD}}{s})(45\,s)^2$

$\boxed{\Delta\theta = 20048 \text{ RAD}}$

Act III: Connecting linear and angular kinematic quantities

RK.2.L2-6:

Description: Units and dimensions of angular velocity. (2 minutes)

Learning Objectives: [1, 12, 13]

Problem Statement: Given that the tangential component of velocity is equal to the angular velocity times the radius, $\mathbf{v}_t = \boldsymbol{\omega}\,\mathbf{r}$, what are the dimensions of $\boldsymbol{\omega}$? The SI units of $\mathbf{v}_t$ are m/s, $\boldsymbol{\omega}$ has SI units of rad/s, and $\mathbf{r}$ has SI units of m.

(1) m/s

(2) [L]/[T]

(3) [Rad]/[T]

(4) 1/[T]

(5) [L]·[Rad]/[T]

RADIANS ARE DIMENSIONLESS

RK.2.L2-7:

Description: Choose the correct mathematical representation for velocity for non-UCM. (3 minutes)

Learning Objectives: [1, 12, 13]

Problem Statement: We have already motivated why we use polar coordinates for an object moving in a circle. More generally, an object can move in a helix for which we would use cylindrical coordinates as follows: $\vec{\mathbf{v}} =< \mathbf{v}_r, \mathbf{v}_t, \mathbf{v}_z >$. Which of the following velocity vectors could represent an object traveling in a circle while changing its speed?

(1) $< 0, 2, \cancel{7} > m/s$

(2) $< \cancel{-3}, 1, 0 > m/s$

(3) $< \cancel{-4}, 0, 0 > m/s$

(4) $< 0, -3, 0 > m/s$

RK.2.L2-8:

Description: Determine relationship between speed of two objects rotating at different radii. (2 minutes + 2 minutes + 3 minutes)

Learning Objectives: [1, 12, 13]

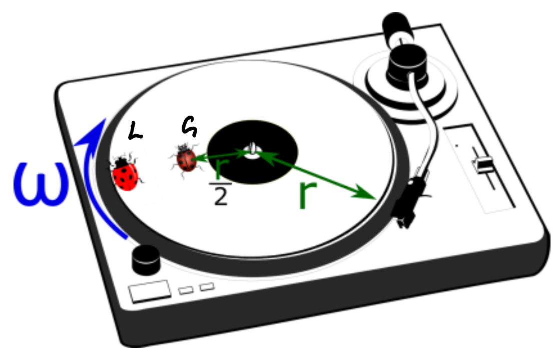
Problem Statement: A ladybug sits at the outer edge of a record player which is spinning clockwise, and a gentleman bug sits halfway between her and the axis of rotation. The record player makes 33 1/3 revolutions every 1 minute.

(a) Is the gentleman bug's speed is _______________ the ladybug's speed.

(1) greater than
(2) less than
(3) equal to

$V_t = \omega r$ * SAME ω

So $V_t \propto r$



(b) The gentleman bug's speed is _____________ the ladybug's speed.

(1) the same as
(2) one quarter of
(3) half of
(4) double
(5) quadruple

$V_t \propto r$ So IF $r \to \frac{1}{2} r$

$V_t \to \frac{1}{2} V_t$

(c) The gentleman bug's radial component of acceleration is _____________ the ladybug's speed.

(1) the same as
(2) one quarter of
(3) half of
(4) double
(5) quadruple

$a_r = \frac{V_t^2}{r}$ OR.. $a_r = \omega^2 r$

$a_r \propto r$

RK.2.L2-9:

Description: Calculate radial and tangential component of acceleration when given angular acceleration, angular velocity and radius. (2 minutes + 2 minutes + 2 minutes)

Learning Objectives: [1, 12, 13]

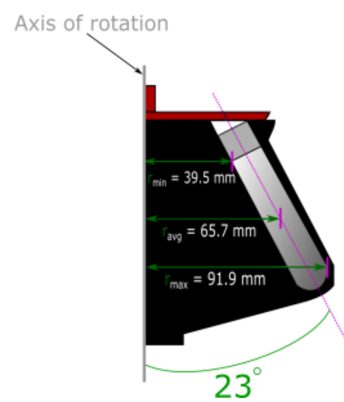
Problem Statement: A Beckman type 70 Ti centrifuge rotor is shown below. The Beckman ultracentrifuge that the rotor is in can accelerate the type 70 Ti rotor with an angular acceleration of 20 rad/s^2. After about 2.5 minutes, the rotor is spinning counter-clockwise at 3,100 rad/s.

(a) What is the radial component of acceleration at the tip of a tube in the type 70 rotor when it reaches 3,100 rad/s?

$$a_r = \frac{v_t^2}{r} \quad \text{OR} \quad a_r = \omega^2 r$$

$$= \left(3100 \frac{\text{RAD}}{\text{s}}\right)^2 \left(91.9 \times 10^{-3} \text{m}\right)$$

$$\boxed{a_r \approx 883000 \frac{\text{m}}{\text{s}^2}}$$



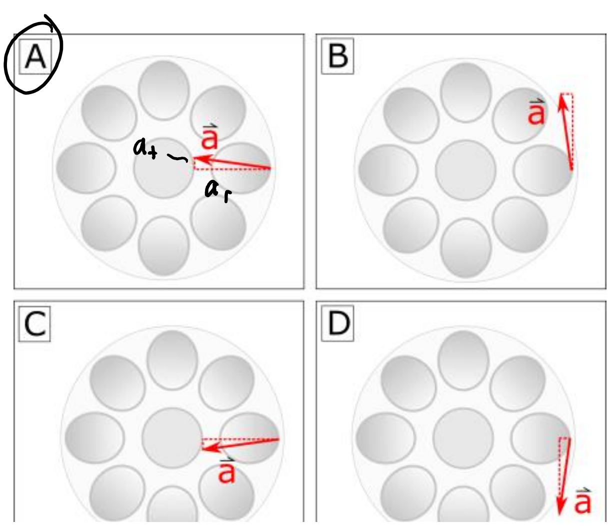
(b) What is the tangential component of acceleration at the tip of a tube in the type 70 rotor when it reaches 3,100 rad/s?

$$a_t = \alpha r$$

$$= \left(20 \frac{\text{RAD}}{\text{s}^2}\right)\left(91.9 \times 10^{-3} \text{m}\right)$$

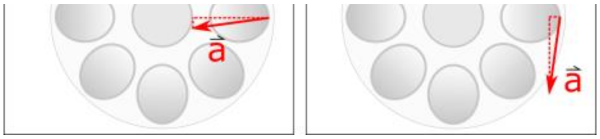
$$\boxed{a_t \simeq 1.84 \frac{\text{m}}{\text{s}^2}}$$

(c) Which of the following acceleration vectors correctly describes the acceleration of the tip of a tube in the type 70.1 Ti rotor when it reaches 3,100 rad/s? Recall the rotor is speeding up counter-clockwise.



$$\vec{a} = \langle a_r, a_t \rangle$$

$$\vec{a} = \langle 883000, 1.84 \rangle \text{ m/s}^2$$



RK.2.L2-10:

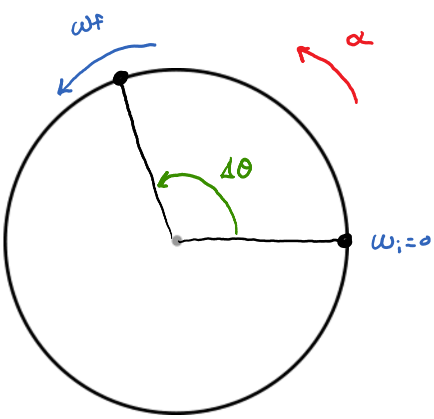
Description: Rotational kinematics problem solving for angular acceleration, linear distance, linear velocity, and linear acceleration. (4 minutes + 4 minutes + 3 minutes + 3 minutes)

Learning Objectives: [1, 12, 13]

Problem Statement: The settings on a Beckman ultracentrifuge is set such that a type 70 Ti rotor starts from rest and uniformly speeds up to 12,500 RPM in 50 seconds.

SI) $12500 \frac{\text{REV}}{\text{MIN}} \times \frac{1\text{MIN}}{60\text{SEC}} = 208\ \text{Hz}$

(a) What is the angular acceleration (in rad/s^2) of the rotor?



$\Delta\theta$ ω_i ω_f α Δt	
K	**UK**
$\omega_i = 0$	$\Delta\theta$
$\omega_f = 2\pi(208\ \text{Hz})$	α
$\Delta t = 50\ s$	

$\Delta\theta = \omega_i\,\Delta t + \frac{1}{2}\alpha\,\Delta t^2$

$\omega_f = \omega_i + \alpha\,\Delta t$

$\omega_f^2 = \omega_i^2 + 2\alpha\,\Delta\theta$

$$\omega_f = \cancel{\omega_i}^{\,0} + \alpha\Delta t$$

$$\alpha = \frac{\omega_f}{\Delta t} = \frac{2\pi(208\ \text{Hz})}{50\ s} \approx \boxed{26.1\ \frac{\text{RAD}}{s^2}}$$

(b) How far (in meters) does the tip of a tube in the rotor travel in the first 50 seconds?

$$\text{ARC LENGTH} \doteq S = \Delta\theta\, r$$

$$S = (32625\ \text{RAD})(91.9 \times 10^{-3}\ m)$$

$$\boxed{S \approx 3000\ m}$$

$$\Delta\theta = \cancel{\omega_i \Delta t}^{\,0} + \frac{1}{2}\alpha\Delta t^2$$

$$\Delta\theta = \frac{1}{2}\left(26.1\ \frac{\text{RAD}}{s^2}\right)(50\ s)^2$$

$$\Delta\theta \approx 32625\ \text{RAD}$$

(c) What is the speed (in m/s) of the tip of a tube in the rotor at the 50 second mark?

(c) What is the speed (in m/s) of the tip of a tube in the rotor at the 50 second mark?

$$V_t = \omega r$$

$$= 2\pi(208\text{ Hz})(91.4\times10^{-3}\text{ m})$$

$$\boxed{V_t = 120\text{ m/s}}$$

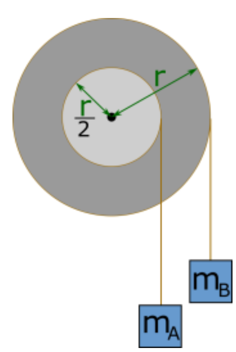
(d) What is the linear acceleration vector of the tip of a tube in the rotor at the 50 second mark?

$$\vec{a} = \langle a_r, a_t\rangle \qquad a_r = \omega^2 r \qquad a_t = \alpha r$$

$$\boxed{\vec{a} = \langle 157000,\ 2.40\rangle\text{ m/s}^2} \qquad \approx 157000\text{ m/s}^2 \qquad \approx 2.40\text{ m/s}^2$$

Conceptual questions for discussion

1. When using any of the three rotational kinematics equations, do you need to use radians for angular displacement, rad/s for angular velocity, and rad/s^2 for angular acceleration?
2. Describe a scenario where the angular position is positive but the angular velocity is negative.
3. Do you agree with the following statement: the area under an angular velocity vs time graph is the angular position?
4. Do you agree with the following statement: the area under an angular acceleration vs time graph is the angular velocity?
5. Sketch all three rotational kinematic quantity vs time graphs for the following scenario: A disk starts at 0 radians, begins to rotate CW slowly, then quickly speeds up in the CCW direction eventually going into the negative radian region.
6. Can a disk have a CCW angular acceleration and a CW angular velocity? If so, describe the motion of the disk as time progresses.
7. Two disks of radius **r** and **r**/2 are concentrically fused together and can freely rotate about their shared center as seen in the image below. Box **A** and box **B** hang from two strings lightly wrapped around each disk. When released from rest, the boxes will begin to fall downward and the strings do no slip relative to the disks they are wrapped around. Which of the following geometric constraints about the distances the boxes travel are?



   i. $|\Delta\vec{r}_A| = |\Delta\vec{r}_B|$

   ii. $|\Delta\vec{r}_A| = 2\,|\Delta\vec{r}_B|$

   iii. $|\Delta\vec{r}_A| = \frac{1}{2}|\Delta\vec{r}_B|$

   iv. $|\Delta\vec{r}_A| = 4|\Delta\vec{r}_B|$

   v. $|\Delta\vec{r}_A| = \frac{1}{4}|\Delta\vec{r}_B|$

   vi. Cannot determine the constraint without knowing the mass of each box.

Hints

RK.2.L2-1: Is 1,500 RPM an angular velocity or frequency?

RK.2.L2-2: No hints.

RK.2.L2-3: No hints.

RK.2.L2-4: No hints.

RK.2.L2-5: Complete the physical representation first to get a better understanding of what is happening. Before doing algebra, use your known and unknown table to determine if there are enough equations with the number of unknowns you have to be able to solve for anything.

RK.2.L2-6: No hints.

RK.2.L2-7: No hints.

RK.2.L2-8: Think proportional reasoning.

RK.2.L2-9: No hints.

RK.2.L2-10: No hints.