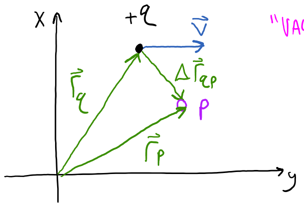
Magnetic fields

Magnetic field from charged point particle

- Biot-Savart law - Empirically derived mathematical representation for how a point charge can create a magnetic field.



$$\vec{B}_p = \frac{\mu_0}{4\pi} q \frac{\vec{v} \times \Delta\hat{r}_{qp}}{|\Delta\vec{r}_{qp}|^2}$$

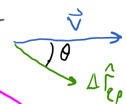
- $\vec{B}_p$: MAGNETIC FIELD AT POINT P
- $\frac{\mu_0}{4\pi}$: THE MAGNETIC FIELD DEPENDS ON THE PROPERTIES OF SPACE "VACUUM PERMEABILITY"
- q: POINT PARTICLE'S CHARGE
- $\vec{v}$: POINT PARTICLES VELOCITY
- $\times$: CROSS PRODUCT
- $\Delta\hat{r}_{qp}$: DIRECTION. POINTS FROM q TO p.
- $|\Delta\vec{r}_{qp}|^2$: DISTANCE BETWEEN POINT PARTICLE AND THE LOCATION OF INTEREST "p"

* UNITS

$$\vec{B} \rightarrow \frac{N}{A\,m} \equiv \text{TESLA} \equiv T$$

$$\mu_0 = 4\pi \times 10^{-7} \frac{T\,m}{A}$$

VECTOR OPERATION



SMALLEST ANGLE BETWEEN $\vec{v}$ AND $\Delta\hat{r}_{qp}$

$$|\vec{B}_p| = \frac{\mu_0}{4\pi} |q| \frac{|\vec{v}| \sin\theta}{|\Delta\vec{r}_{qp}|^2}$$

* W/ MAGNITUDE OF $\vec{B}$, USE RIGHT HAND RULE TO FIND DIRECTION

RHR FOR CROSS PRODUCTS

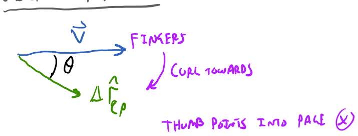
1. FINGERS IN DIRECTION OF 1ST VECTOR ($\vec{v}$)
2. CURL FINGERS TOWARDS 2ND VECTOR ($\Delta\hat{r}_{qp}$)
3. THUMB POINTS IN DIRECTION OF RESULTANT ($\vec{B}$)

   * RESULTANT IS ⊥ TO BOTH 1ST + 2ND VECTORS

* CONVENTION
  - ⊗ INTO PAGE
  - ⊙ OUT OF PAGE

RHR EXAMPLE:

VECTOR OPERATION

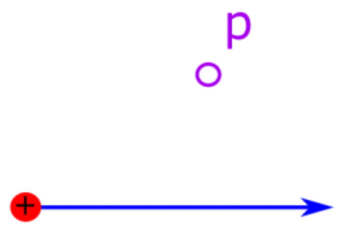


PRACTICE: Consider a point particle with positive charge and velocity shown below. What is the direction of the magnetic field at point p?

1. Up

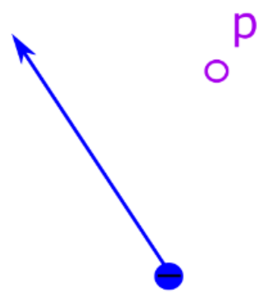
direction of the magnetic field at point p?

1. Up
2. Down
3. Left
4. Right
5. Into page
6. Out of page



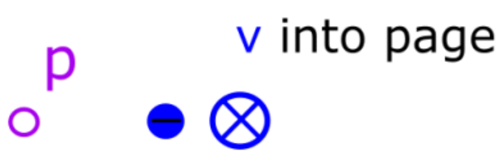
PRACTICE: Consider a point particle with negative charge and velocity shown below. What is the direction of the magnetic field at point p?

1. Up
2. Down
3. Left
4. Right
5. Into page
6. Out of page



PRACTICE: Consider a point particle with negative charge and velocity shown below. What is the direction of the magnetic field at point p?

1. Up
2. Down
3. Left
4. Right
5. Into page
6. Out of page

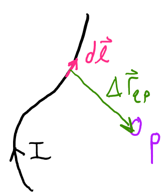


PRACTICE: Consider a point particle with positive charge and velocity out of the page. What is the direction of the magnetic field at point p?

1. Up
2. Down
3. Left
4. Right
5. Into page
6. Out of page

Magnetic field from current carrying wire

- Biot-Savart law - Empirically derived mathematical representation for how a point charge can create a magnetic field.



$$d\vec{B}_p = \frac{\mu_0 I}{4\pi} \frac{d\vec{\ell} \times \Delta\hat{r}_{\ell p}}{|\Delta\vec{r}_{\ell p}|^2}$$

VACUUM PERMEABILITY

CURRENT

LINE ELEMENT

DIRECTION. POINTS FROM LINE ELEMENT $d\ell$ TO POINT P

THE "TINY BIT OF" MAGNETIC FIELD AT POINT P CREATED BY A TINY LINE ELEMENT WITH CURRENT I.

DISTANCE BETWEEN LINE ELEMENT AND THE LOCATION OF INTEREST "P"

$$\vec{B}_p = \int d\vec{B}_p$$

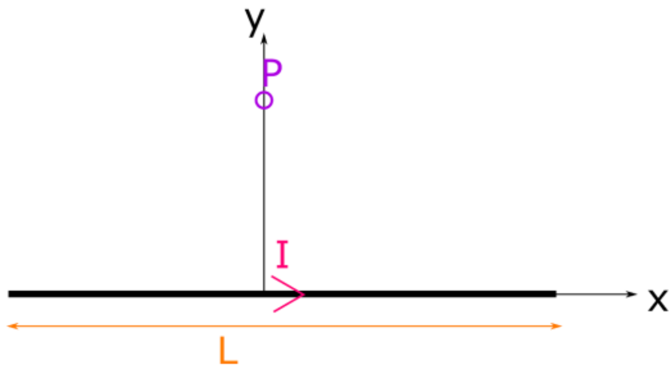
THE MAGNETIC FIELD AT POINT P

$$|d\vec{B}| = \frac{\mu_0 I}{4\pi} \frac{d\ell \sin\theta}{|\Delta\vec{r}_{\ell p}|^2}$$

SMALLEST θ BETWEEN $d\vec{\ell}$ AND $\Delta\hat{r}_{\ell p}$ WHEN PLACED TAIL TO TAIL

* USE RHR FOR CROSS PRODUCTS TO FIND DIRECTION → $d\vec{\ell} \times \Delta\hat{r}_{\ell p}$

PRACTICE: Consider a thin straight wire with current I as shown in the figure below. Let's find the magnetic field at point P.



What is the line element that carries current I in this wire?

1. dx $\hat{x}$
2. dy $\hat{y}$
3. dz $\hat{z}$
4. R dθ $\hat{\theta}$

What is the displacement vector that points from the line element to point P?

1. < x, y, 0 >
2. < L/2, y, 0 >
3. < x, L/2, 0 >
4. < 0, y, z >

Rewrite sinθ as a function of x, y, z.

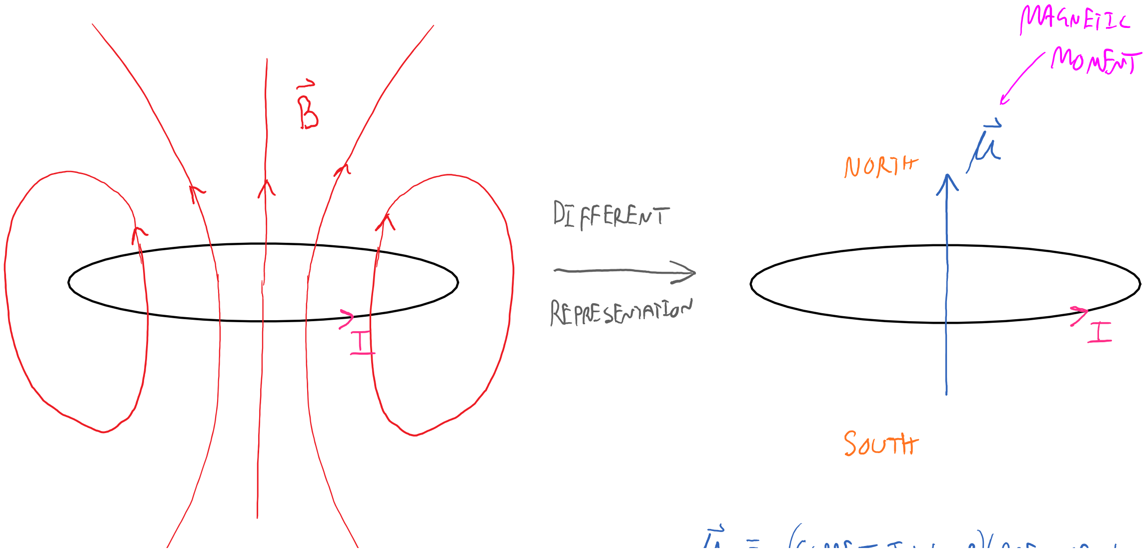
1. $x / (x^2 + y^2)^{1/2}$
2. $y / (x^2 + y^2)^{1/2}$
3. $z / (x^2 + y^2)^{1/2}$

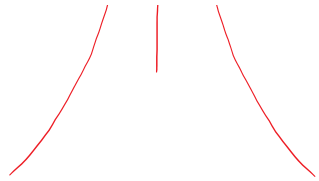
Use the RHR to find the direction of the magnetic field at point P.

1. Into page.
2. Out of page.
3. x-direction
4. y-direction
5. z-direction

Construct the integral to find the magnetic field at point P.

Magnetic field from a current loop





$\vec{\mu}$ ≡ (CURRENT IN LOOP)(AREA OF LOOP) IN DIRECTION OF $\vec{B}$

ALTERNATIVE RHR FOR CURRENT LOOPS

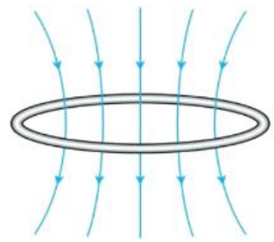
① FINGERS CURL IN DIRECTION OF I

② THUMB POINTS IN DIRECTION OF $\vec{B}$

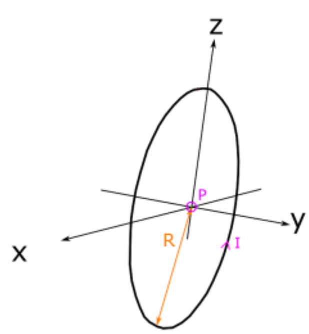
* NORTH AND SOUTH USED TO DETERMINE DIRECTION OF $\vec{\mu}$ AND $\vec{B}$
* NORTH SIDE OF LOOP ATTRACTS SOUTH SIDE OF ANOTHER LOOP
* SOUTH SIDE OF LOOP ATTRACTS NORTH SIDE OF ANOTHER LOOP
* NORTH SIDE OF LOOP REPELS NORTH SIDE OF ANOTHER LOOP
* SOUTH SIDE OF LOOP REPELS SOUTH SIDE OF ANOTHER LOOP

PRACTICE: What is the current direction in this loop as viewed from above? And which side of the loop is the north pole?

1. Current counterclockwise, north pole on bottom.
2. Current counterclockwise, north pole on top.
3. Current clockwise, north pole on bottom.
4. Current clockwise, north pole on top.



PRACTICE: Consider a thin wire with current I bent into a circle as shown in the figure below. Let's find the magnetic field at point P.



What is the line element that carries current I in this wire?

1. dx $\hat{x}$
2. dy $\hat{y}$
3. dz $\hat{z}$
4. R dθ $\hat{\theta}$

What is the magnitude of the displacement vector that points from the line element to point P?

1. R
2. R^2
3. y
4. z

Use the RHR to find the direction of the magnetic field at point P.

1. Into page.
2. Out of page.
3. x-direction
4. y-direction
5. z-direction

Construct the integral to find the magnetic field at point P.