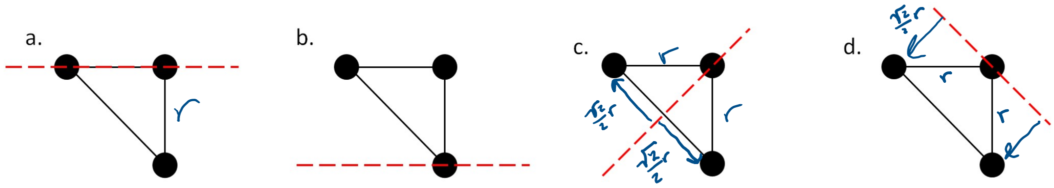
Name: ______________________ ID: ________ ~~Lab~~ (day/time): ____________

Physics 202
Midterm Exam 1
1/23/2019

Collaboration is not allowed. Allowed on your desk are: up to ten 8.5 x 11 inch doubled sided sheets of notes that are bound together, non-communicating scientific calculator, 1 page of scratch paper, writing utensils, and the exam. You will have 80 minutes to complete this exam.

$I_{pm} = mr^2$

1. **(4 points)** Three identical point masses are attached via massless rods as shown in the diagrams below. This system is rotated about the dashed lines shown in each case. Rank the moments of inertia for each orientation. Rank them from smallest to largest, indicating if any are equal. (example: $I_a = I_b < I_c < I_d$ which is not the correct answer!)



Ranking: $I_a = I_c = I_d < I_b$

$I_a = mr^2$

$I_d = I_c = 2\times\left(\frac{\sqrt{2}}{2}r\right)^2 m = 2\times\frac{2}{4}r^2 m = \frac{1}{2}mr^2 \times 2 = mr^2$

$I_b = 2\times mr^2 = 2mr^2$

For questions 2 through 4 **fill in the square** next to all correct answers, a given problem may have more or less than one correct answer. Each correctly chosen answer will receive two points. There are 7 correct answers in this section and only the first 7 filled in answers will be graded. There is no partial credit.

2. A disk is spinning with an angular velocity ω and an angular acceleration α. For which of the following cases would the disk rotation be slowing down?

a. α is negative, ω can be positive or negative

b. α is positive, ω can be positive or negative

c. ω is negative, α can be positive or negative

d. ω is positive, α can be positive or negative

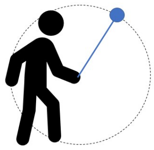
e. α is negative, ω is negative

f. α is negative, ω is positive

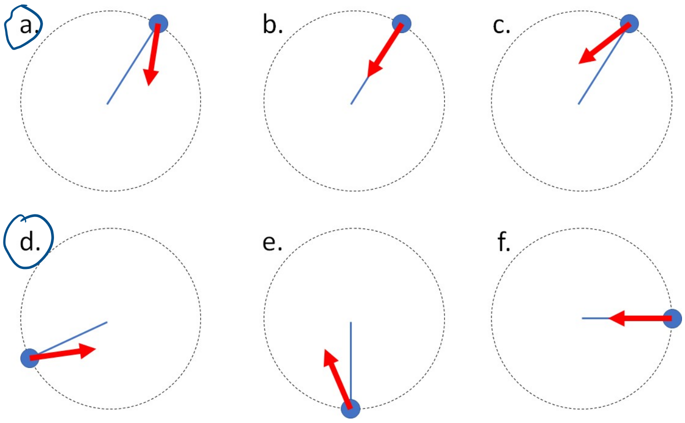
g. α is positive, ω is negative

h. α is positive, ω is positive

3. Yo-Yo Pa is back at it, swinging his yoyo "around the world" counterclockwise in a vertical circle as shown to the right. He provides no torque and just holds his hand perfectly in place. Which of the following diagrams could accurately describe the linear acceleration of the yoyo?

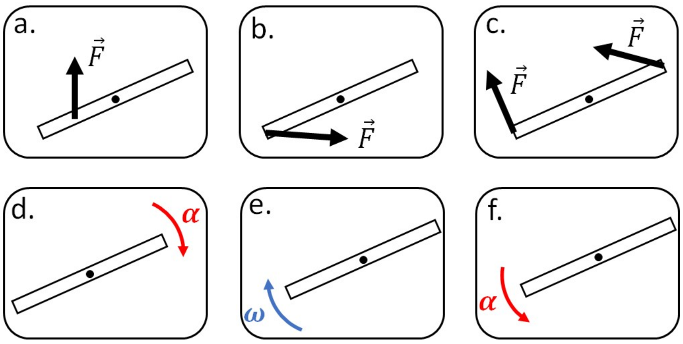


a.
b.
c.
d.
e.
f.

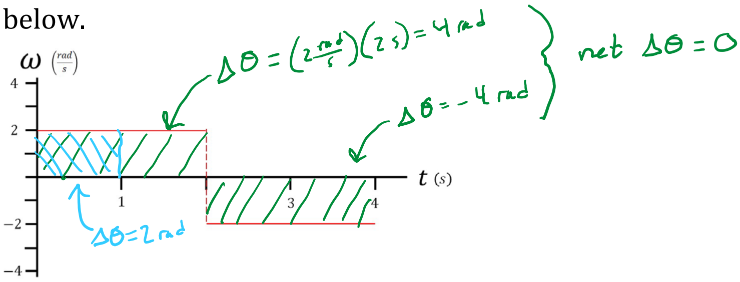


4. For which of the following cases must there *necessarily* be a negative net torque on the bar about its center of mass? All indicated forces have the same magnitude.

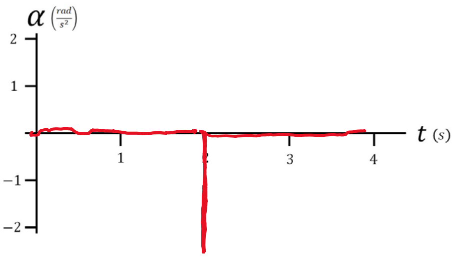
a.
b.
c.
d.
e.
f.



5. A disc starting at $\theta = 0\ rad$ spins with an angular velocity described by the plot below.

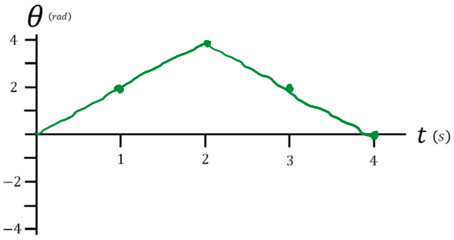


a. (3 points) Draw the disc's angular acceleration on the plot below.



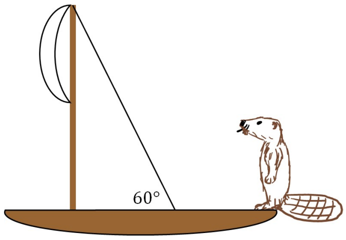
α => slope of ω

b. (3 points) Draw the disc's angular position on the axes below.



Δθ => area under ω

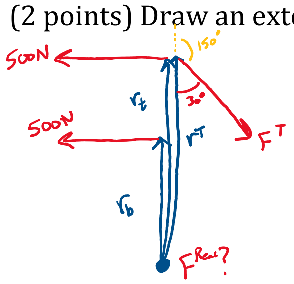
6. Benny has taken to the high seas on his ship. Wind catches the main sail and exerts a total force of 1000 N. This force is divided evenly between the two locations where the sail is tied to the mast (the central pole on a ship). The sail is tied to the top of the 5.0 meter tall mast and again 2.0 meters from the top. There is a rope tied to the top of the mast and anchored to the deck of the boat as pictured.

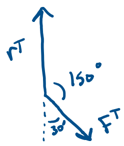


a. (1 point) At what location would you choose your reference axis so that you can solve for the tension in the rope?

base of mast b/c lever arm of reaction force will then be 0 ⇒ $\tau = 0$

b. (2 points) Draw an extended free body diagram (eFBD) for the mast.





c. (3 points) Find the tension in the string.

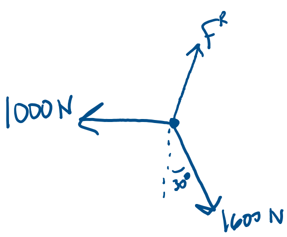
$$\tau_{net} = 0 = (500\,N)(5m) + (500\,N)(3m) - F^T(5m)\sin(150)$$

$$0 = 2500\,Nm + 1500\,Nm - F^T \frac{5m}{2}$$

$$F^T = \frac{4000\,Nm}{\frac{5}{2}m} = 1600\,N$$

d. (3 points) Find the reaction force from the deck on the bottom of the mast.

FBD

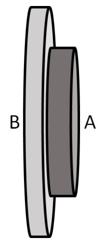


$$F^R_y = 1600\cos(30°) = 1390\,N$$

$$F^R_x = -1600\sin(30°) + 1000 \approx 200\,N$$

$$F^R = \langle 200, 1390 \rangle\,N$$

7. A space station is made of two disks stacked flat side to flat side as pictured. Both disks have identical mass. Disk B has a larger radius than disk A. They are attached only at the center of the disks via a shared axis of rotation and can rotate separately. Before the mice overlords can colonize the station, the disks must be spun up to speed to generate artificial gravity. The disks begin from rest. Using internal motors, the disks apply torques of equal magnitude, but opposite direction on each other (Newton's 3rd law says they must be!). Thus, the disks begin spinning in opposite directions. No torques are applied by any external sources.



No credit will be given without explanation. You are encouraged to use any combination of diagrams, mathematical relations, and words in your explanations!

a. (2 points) After a few minutes, which disk, if either, is spinning faster? Explain.

$I_B > I_A$

$|\tau_A| = |\tau_B|$

$\tau = I\alpha$

$\Rightarrow \alpha_A > \alpha_B$

greater I w/same τ

$\Rightarrow$ smaller α

b. (2 points) Which disk, if either, has a larger angular momentum magnitude? Explain.

no external $\tau \Rightarrow \Delta L = 0$

$L_i = 0 = L_f = L_A + L_B$

$\Rightarrow L_A = -L_B$

$\Rightarrow |L_A| = |L_B|$

c. (2 points) Which disk, if either, has more rotational kinetic energy? Explain.

$I_A \omega_A = I_B \omega_B$

$\omega_A > \omega_B$

$\Rightarrow \frac{1}{2} I_A \omega_A^2 > \frac{1}{2} I_B \omega_B^2$

$KE_{rot,A} > KE_{rot,B}$

8. Question 8 asks only about disk B. Disk B from the mice's space station has an initial angular acceleration of $2.25 \times 10^{-4} \; \frac{rad}{s^2}$ and a radius of 111 meters.

α $\quad$ r

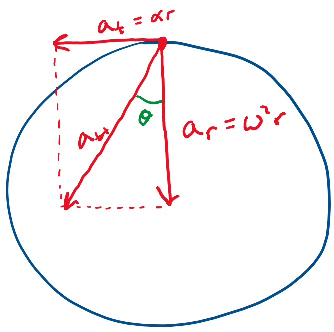
a. (2 points) What is the angular velocity of disk B after 1.11 minutes?

$$(1.11 \text{ min}) \, 60 = 66.7 \text{ sec}$$

$$\omega_f = \omega_i^{\,0} + \alpha \Delta t$$

$$= \left(2.25 \times 10^{-4} \frac{rad}{s^2}\right)(66.7 s) = 0.015 \frac{rad}{s}$$

b. (6 points) After 66.7 seconds, what is the angle between the linear acceleration vector and the radial direction?



$$\tan\theta = \frac{a_t}{a_r} \qquad a_t = \alpha r \qquad a_r = \omega^2 r$$

$$\tan\theta = \frac{\alpha r}{\omega^2 r} = \frac{\alpha}{\omega^2} = \frac{2.25 \times 10^{-4}}{(0.015)^2} = 1 \Rightarrow \boxed{\theta = 45^\circ}$$