Magnetic force

Select LEARNING OBJECTIVES:

- Explain how magnetic fields can be created without current.
- Use superposition to find the magnetic field at a location when multiple magnets are present.

TEXTBOOK CHAPTERS:

Boxsand :: uniform circular motion in magnetic fields

WARM UP: Magnetic field questions?

In the introduction to magnetic fields there is a walkthrough of an experiment with a positive point charge and a current carrying wire. Take a moment to review that experiment. In addition to the surprise of needing to identify a new type of force, the magnetic force, it was observed that this magnetic force is only present on the positive point particle when the particle had a velocity. In this lecture we will explore the magnetic force on charged particles in the presence of a magnetic field, as well as the magnetic force on current carrying wires in the presence of a magnetic field. Before we dive into finding the magnetic force, it's beneficial if we briefly review the field model in terms of magnetic fields and how they relate to magnetic forces.

Review of the field model

We have already explored how magnetic fields are created: by charged point particles with a velocity, by current carrying wires, and by inherently magnetic materials via quantum mechanics. Remember that the essence of the field model is that something creates a field, and then something else that finds itself in that field experiences some force associated with the interaction with the field. In terms of magnetic fields, let's assume that a current carrying wire creates a magnetic field everywhere in space around it. Then a charged point particle that moves with velocity within the magnetic field created by the wire feels a magnetic force due to the interaction between itself and the magnetic field. Even if the charged point particle didn't exist, the magnetic field from the wire would still be present, just no magnetic forces exists until the charged point particle enters the magnetic field with a velocity.

Magnetic force on moving point particle with charge

Through experiments, one can show that the magnetic force on a charged point particle with velocity turns out to be perpendicular to both the velocity and the magnetic field. This type of perpendicular behavior invites us to use the cross product, a vector operation, to mathematically model the magnetic force. The mathematical model is shown below.

CROSS PRODUCT

$$\boxed{\vec{F}^{B}_{\text{on } q_o} = q_o \vec{v} \times \vec{B}}$$

EXTERNAL MAGNETIC FIELD THAT POINT PARTICLE IS MOVING THROUGH

MAGNETIC FORCE ON q_o

CHARGE OF POINT PARTICLE

VELOCITY OF POINT PARTICLE

SMALLEST ANGLE BETWEEN $\vec{v}$ AND $\vec{B}$ WHEN PLACED TAIL-TO-TAIL

* IN PRACTICE:

- $|\vec{F}^{B}| = |q_o| |\vec{v}| |\vec{B}| \sin\theta$
- THEN USE RHR FOR CROSS PRODUCTS TO FIND DIRECTION

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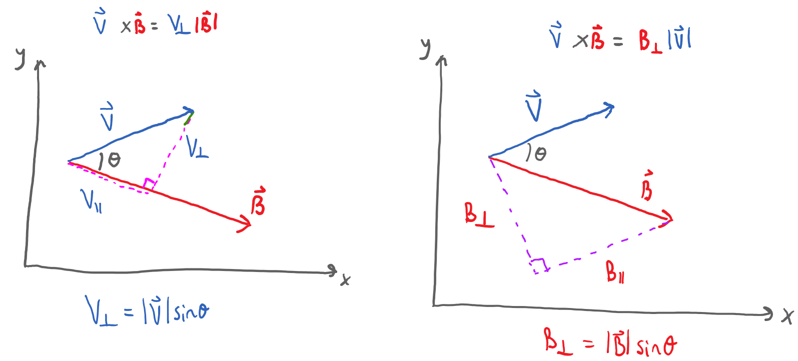
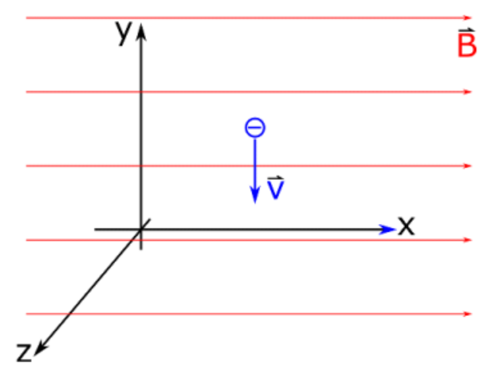
* RECALL: CROSS PRODUCTS "ASKS HOW $\perp$ TWO VECTORS ARE TO EACH OTHER".

* THE MORE $\perp$ $\vec{V}$ AND $\vec{B}$ ARE, THE LARGER THE MAGNETIC FORCE



Here we wish to find the magnetic force on a charged point particle. Force is a vector thus we must rely on our new skills of finding a vector when cross products are involved. Recall that we first find the magnitude of the vector, then use the right hand rule to find the direction. It's also useful to think in general what the cross product is doing; in essence it is asking how perpendicular two vectors are to each other. When the cross product is its maximum value, the two vectors are perpendicular to each other, when the cross product is zero the two vectors are parallel to each other.

PRACTICE: An electron moves along the *-y* axis with a speed of 1.0×10^7 *m/s*. A *0.50 T* magnetic field points in the positive *x-direction*. What is the force on the electron?



1. 8×10^{-13} N, negative z-direction
2. 8×10^{-13} N, positive z-direction
3. 7×10^{-12} N, negative x-direction
4. 7×10^{-12} N, negative y-direction
5. 6×10^{-11} N, negative z-direction

PRACTICE: A beam of electrons enters a region with a magnetic field as shown below. If the beam is deflected upwards, the magnetic field must be oriented

1. downwards

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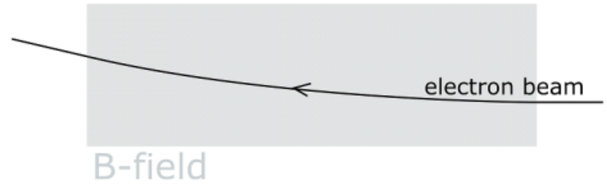
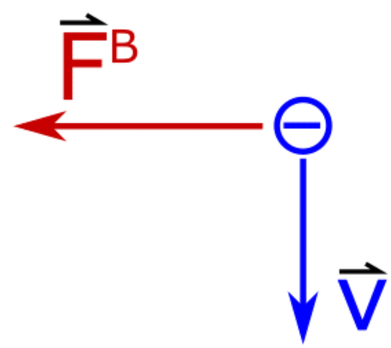
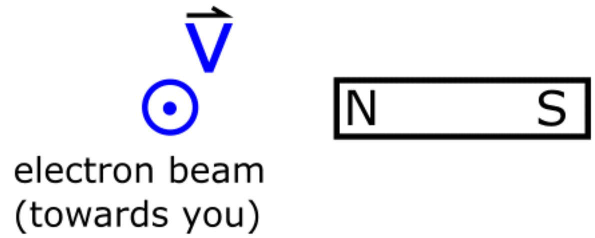


1. downwards.
2. up.
3. into the plane of the drawing.
4. out of the plane of the drawing.
5. to the left.
6. to the right.
7. None of the above -- it is at an angle.
8. Need more information to determine.

PRACTICE: An electron moves perpendicular to a magnetic field. Sketch a vector to represent the direction of the magnetic field.

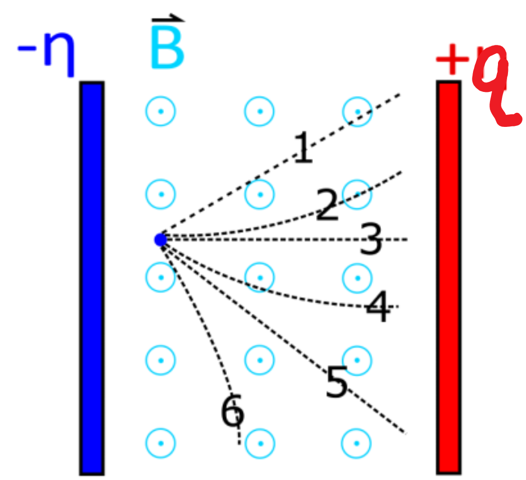


PRACTICE: A beam of electrons moves towards you perpendicular to your screen. The north pole of a permanent bar magnet is brought near the beam, pointing towards the beam. Indicate the direction of the magnetic force on the electrons.



A negatively charged particle is released from rest between the plates of a capacitor under the combine influence of a magnetic field B (directed out of the page) and the electric field inside the capacitor. Which of the paths shown best represents the trajectory of the particle (ignore gravity)?

1. 1
2. 2
3. 3
4. 4
5. 5
6. 6
7. The particle remains at rest
8. The particle moves out of the plane of the drawing.



Magnetic force on current carrying wires

As stated in the introduction to this lecture, it is observed that a current carrying wire experiences a force in the presence of an external magnetic field. The mathematical model requires the use of a cross product again. This time the cross product is between the length vector and the external magnetic field. The expression for the magnetic force on a current carrying wire in the presence of an external magnetic field is shown below.

$$\boxed{\vec{F}^B = I\,\vec{\ell} \times \vec{B}}$$

LENGTH VECTOR OF THE WIRE THAT IS CARRYING THE CURRENT

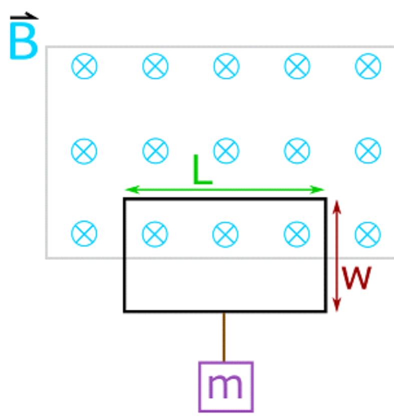
* IN PRACTICE

- $|\vec{F}^B| = I\,|\vec{\ell}|\,|\vec{B}|\sin\theta$
- THEN USE RHR FOR DIRECTION

Recall that current is not a vector. This seems like a weird statement because we often talk about the direction of current. This is where our definition of a vectoring being something with a magnitude and direction begins to fail. Vectors are defined more precisely based off of how their components change under transformations of coordinate systems. This is above and beyond the scope of this class, but it is useful to bring up because you might notice that current does not have a vector over top of it, the length of the wire does. It turns out if we measure the length of the wire with a rule, and assign a direction to it, then it behaves property based off the more formal definition of vectors. The direction that this length vector has is in the direction of the current that flows through that segment of wire.

PRACTICE: A rectangular loop of wire, supporting mass m, hangs vertically with one end in a uniform magnetic field B which points into the page as shown in the figure below.

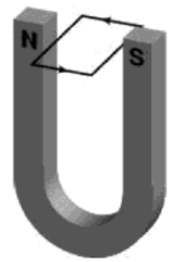
Which direction should current flow to make the net magnetic force upwards?



For what current, I, in the loop would the magnetic force cause the mass to be in static equilibrium?

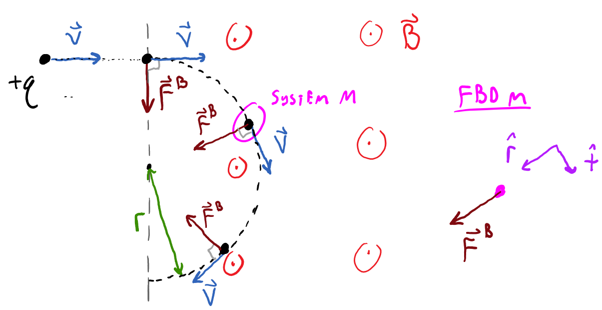
PRACTICE: A current loop is placed between the poles of a horseshoe magnet, as shown below. The loop tends to

1. rotate, left side up.
2. rotate, right side up.
3. rotate, front side up.
4. rotate, rear side up.
5. none of the above -- it stays in place.
6. other.



Uniform circular motion of a charged particle in an external magnetic field

We already explored the magnetic force on a charged point particle with velocity in an external magnetic field. We even looked at the trajectory of this charged point particle. However, when the external magnetic field is uniform and perpendicular to the velocity, the motion of the charged particle exhibits uniform circular motion. If the velocity is not perpendicular to the magnetic field, the motion would be a spiral shape. The pressing question is, why is the motion a perfect circle? Recall that the magnetic force is perpendicular to the velocity. Thus the magnetic force will initially cause the particle to change its velocity, but as the velocity changes the magnetic force stays perpendicular. This process results in the magnetic force always pointing inwards towards the center of curvature. With no other forces on the charged particle, the motion is uniform circular motion. Now that you know the motion will be UCM, you should apply a force analysis to the particle and see what it results in. The analysis is done below, but I strongly encourage you to start from scratch and go through the force analysis process.



$$\sum \vec{F}_{m_q} = m\vec{a}$$

$$\sum F_r = m a_r$$

$$|\vec{F}^B| = m\frac{v_T^2}{r}$$

$\theta = 90^\circ$

$$|q||\vec{v}||\vec{B}|\cancel{\sin\theta}^{\,1} = m\frac{v_T^2}{r}$$

$$q v_T B = \frac{m v_T^2}{r}$$

$$\boxed{qB = \frac{m v_T}{r}}$$

TANGENTIAL COMPONENT OF VELOCITY

An interesting result from the analysis above is how the radius of the circular motion is dependent on the mass of the particle, the charge of the particle, the external magnetic field strength, and velocity of the particle. I suggest solving for the radius to explicitly see its functional dependence on those variables. Try to convince yourself that each variable affects the radius how you would expect. Some intuition and spidey senses are required which why it makes for a good exercise.

PRACTICE: Two beams of particles enter the same magnetic field with the same velocity, perpendicular to the field. If the mass of particles in beam A is twice that of B and the charge of particles in A are four thirds that of B. What is the ratio of the radius of curvature of A to that of B?

1. 1/3
2. 2/3
3. 3/4
4. 1
5. 4/3
6. 3/2
7. 3

PRACTICE: Cosmic rays (atomic nuclei stripped bare of their electrons) would continuously bombard Earth's surface if most of them were not deflected by Earth's magnetic field. Given that Earth is, to an excellent approximation, a magnetic dipole, the intensity of cosmic rays bombarding its surface is greatest at the

1. poles.
2. mid-latitudes.
3. equator.