

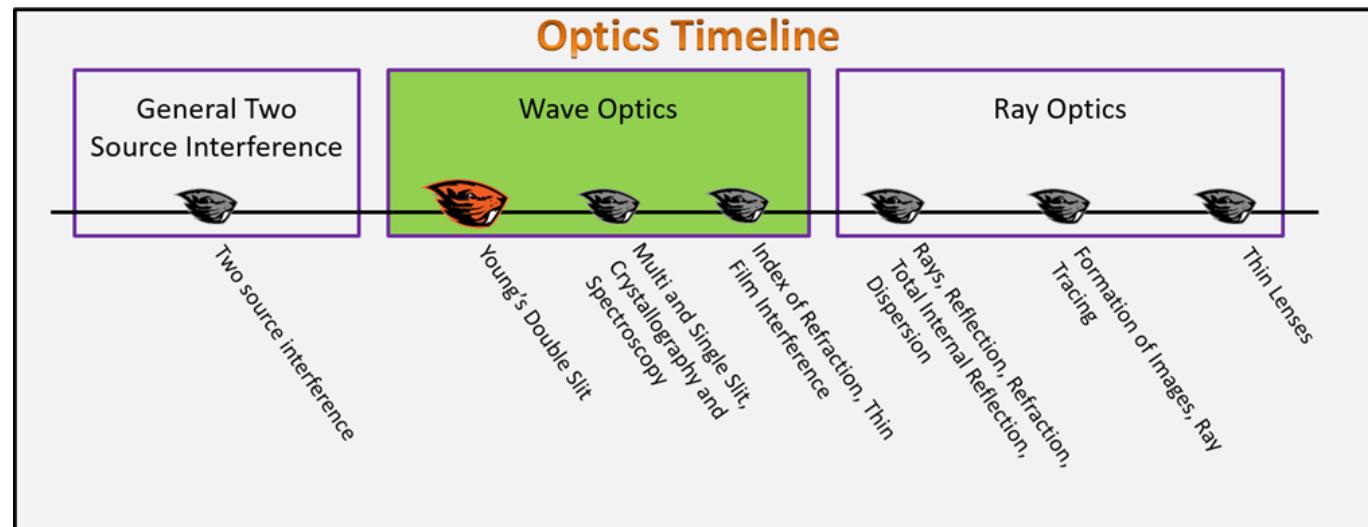
(WO.L1.2-sols) Foundation Stage Solutions

Monday, January 22, 2018 5:44 PM

Wave Optics

Foundation Stage (WO.L1.2)

Lecture 1 Young's Double Slit



Textbook Chapters (* Calculus version)

- **BoxSand** :: KC videos ([Optics](#))
- **Knight** (College Physics : A strategic approach 3rd) ::
- ***Knight** (Physics for Scientists and Engineers 4th) ::
- **Giancoli** (Physics Principles with Applications 7th) ::

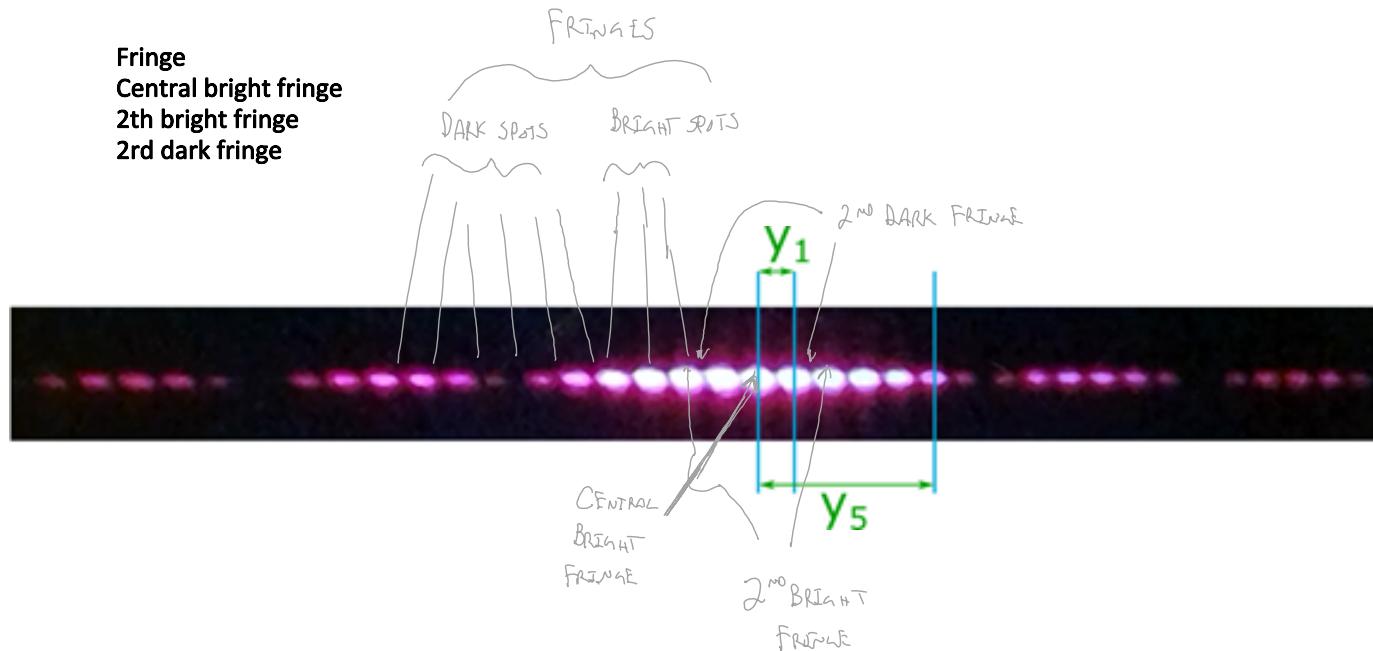
Warm up

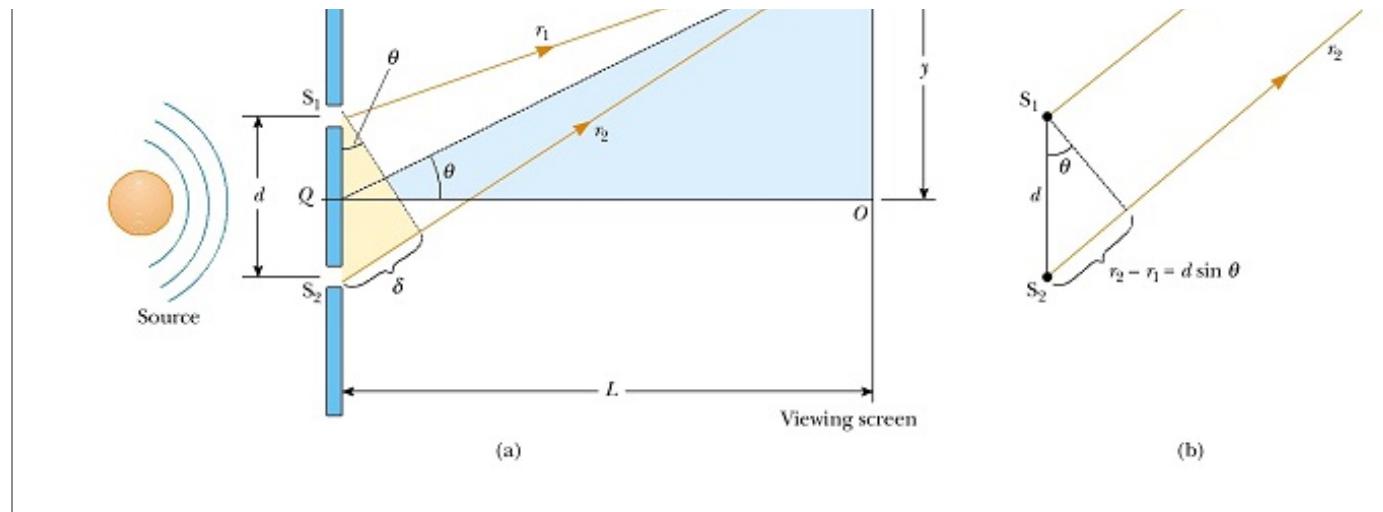
WO.L1.2-01:

Description:

Learning Objectives: [?] - Can you identify the objectives from the previous lecture, and this lecture, that this question is relevant to?

Problem Statement: With your neighbors discuss the following terms with regards to the image below.





Selected Learning Objectives

1. Coming soon to a lecture template near you.

Key Terms

- Path length difference
- Small Angle Approximation
- Slit Separation
- Viewing Screen
- Coherent
- Constructive
- Destructive

Key Equations

$m\lambda = d \sin \theta$	$\tan \theta = \frac{y_m}{L}$
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Questions

Act I: Applications of the Mathematical Model

WO.L1.2-02:

Problem Statement: Young's double slit experiment is just one example of two source interference. Which of the following is/are necessary to see a spatial interference pattern on a screen opposite two slits?

- (1) A single frequency source.
- (2) A coherent source of light.
- (3) Experimental apparatus needs to be in air.
- (4) The source needs to be a light source.

WO.L1.2-03:

Problem Statement: For the double slit experiment, we use a mathematical model $m\lambda = d \sin(\theta)$.

Where in this model is the path length difference?

- (1) m
- (2) λ
- (3) d
- (4) $d \sin(\theta)$
- (5) $\sin(\theta)$
- (6) λd

WO.L1.2-04:

Problem Statement: On observing a double slit experiment, you measure the distance from the slits to the screen to be 1.0 m. You also measure the distance between the central bright spot and the 3rd maximum to be 55 cm. You read on the device that the slits are a distance 4.36 μm apart.

(a) At what angle theta from the central maximum does the 3rd maximum appear?

$$ds \sin \theta_n = m \lambda$$

$$\cancel{?} \quad ?$$

$$ds \sin \theta_3 = 3 \lambda$$

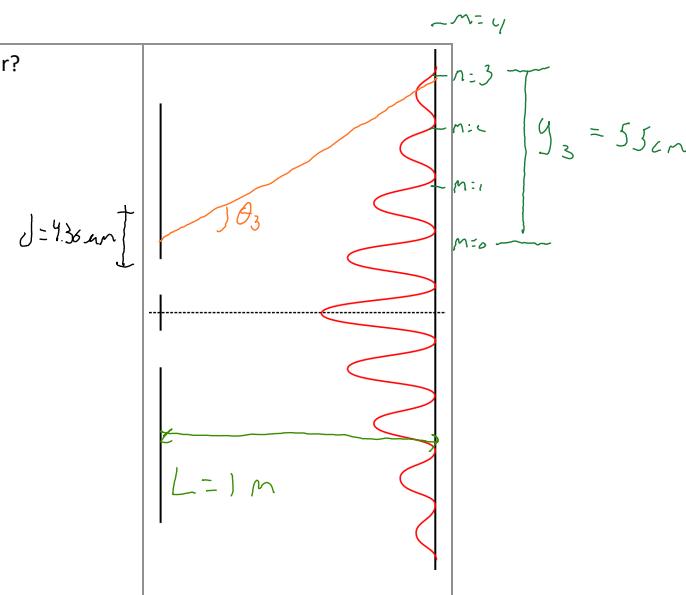
$$y_n = L \tan \theta_n$$

$$\cancel{?} \quad ?$$

$$y_3 = L \tan \theta_3$$

$$\theta_3 = \tan^{-1} \left(\frac{y_3}{L} \right)$$

$$\boxed{\theta_3 \approx 28.8^\circ}$$



(b) Find the wavelength of the laser, λ .

$$(4.36 \times 10^{-6}) \sin(26.8) = 3\lambda$$

$$\lambda \approx 700 \text{ nm}$$

WO.L1.2-05:

Problem Statement: On observing a double slit experiment, you measure the distance from the slits to the screen to be 1.0 m. You read on the device that the slits are a distance 11.2 μm apart. You also read on the laser that the wavelength it emits is 700 nm. We want to find the distance between the 2nd and 3rd dark fringes on the screen.

$$\text{DOUBLE SLIT} \rightarrow \text{DARK SPOTS} \rightarrow p_{LD} = \left(m + \frac{1}{2}\right)\lambda$$

(a) What is the distance y_2 for the 2nd dark fringe?

$$ds \sin \theta = \left(m + \frac{1}{2}\right)\lambda \quad y_1 = L \tan \theta$$

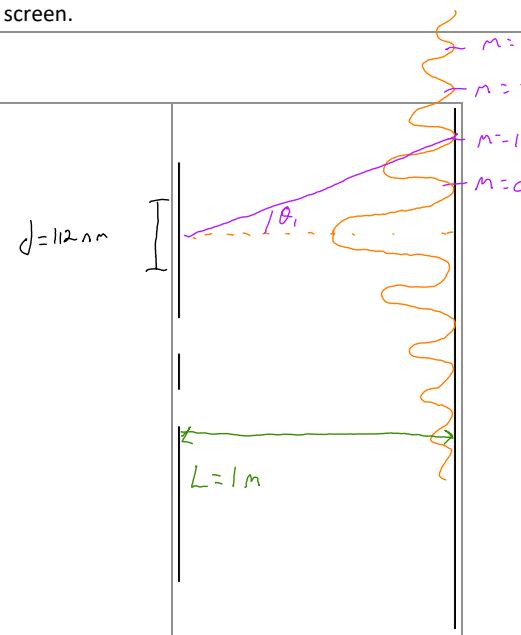
$$ds \sin \theta_1 = \left(1 + \frac{1}{2}\right)\lambda \quad y_1 = L \tan \theta_1$$

$$(11.2 \times 10^{-6}) \sin \theta_1 = \frac{3}{2} (700 \times 10^{-9}) \quad y_1 = (1) \tan(\theta_1)$$

$$\theta_1 \approx 5.37938^\circ$$

$$y_1 \approx 9.42 \text{ cm}$$

$$ds \sin \theta_2 = \left(2 + \frac{1}{2}\right)\lambda \quad y_2 = L \tan \theta_2$$



(b) What is the distance y_3 for the 3rd dark fringe?

$\theta \approx 8.9893^\circ$

$y_3 = 15.8 \text{ cm}$

$$y_2 - y_1 \approx 6.38 \text{ cm}$$

(c) What is the distance from the 2nd to the 3rd dark fringe on the screen?

Act II: Small Angle Approximation

WO.L1.2-06:

0.06099444833... 0.0010000003 0.9999995

0.09985334 0.100334721 0.995004653

0.198663308 0.2027100355 0.9800665779

0.2955204067 0.3093362496 0.9563364691

Problem Statement: Work together in teams to complete the following table.

θ (deg)	θ (rad)	$\sin(\theta)$	$\tan(\theta)$	$\cos(\theta)$
0.0573	0.001	0.0009999833	0.0009999833	0.9999999999
5.73	0.1	0.08414709848	0.08414709848	0.9950046533
11.5	0.2	0.198663308	0.198663308	0.9800665779
17.2	0.3	0.2955204067	0.2955204067	0.9563364691
28.6	0.5	0.44638860257	0.44638860257	0.8775815619
57.3	0.1	0.8414709848	0.8414709848	0.5463024848
114.6	0.2	0.198663308	0.198663308	0.9800665779
171.9	0.3	0.2955204067	0.2955204067	0.9563364691
343.8	0.6	0.64638860257	0.64638860257	0.4226182619
687.6	1.2	0.98160129596	0.98160129596	0.1736481777
1375.2	2.4	0.99999999999	0.99999999999	0.01736481777

FOR "SMALL ANGLE" 0.4

How small? ... $y_0 \propto \theta$

I'm OK with $\theta \leq 1^\circ$

(a) Which of the following relationships are true for some portion of the above table?

- (1) $\sin \theta = \tan \theta$
- (2) $\sin \theta = \cos \theta$
- (3) $\cos \theta = \tan \theta$
- (4) $\theta = \tan \theta$
- (5) $\theta = \sin \theta$
- (6) $\theta = \cos \theta$



(b) For what angles are these equations valid?



NOTE ... $5^\circ \approx \sin(5^\circ)$

(c) Are these equations useful when using degrees? Radians?

WO.L1.2-07:

Problem Statement: When observing a double slit interference pattern, you note that it is in a regime where the small angle approximation is a safe approximation to make.

All θ 's

small θ

Which of the following relationships can you assume to be true?

- (1) $\lambda \gg d$
- (2) $\lambda \ll d$
- (3) $L \gg y_m$
- (4) $L \ll y_m$
- (5) $m \gg d$
- (6) $m \ll d$

$$\theta_m \approx \frac{m\lambda}{d}$$

Small ... so

$$\frac{y_m}{L} \approx \theta_m$$

small

so ..

$$\lambda \ll d$$

$$y_m \ll L$$

WO.L1.2-08 CHECK IF SMALL θ

$$\frac{\lambda}{d} \stackrel{?}{=} \text{SMALL}$$

Problem Statement: When observing a double slit interference pattern for 500 nm wavelength light incident on slits with a separation of around 1 mm, you notice that the distance ~~between~~^{between} the 3rd and 4th order bright spots is 5 mm.

If the screen is 15 meters from the slits, what is the precise slit separation? Small

$$d\theta_n \approx m\lambda$$

$$y_n \approx L\theta_n$$

$$\theta_n \approx \frac{m\lambda}{d}$$

$$\theta_n = \frac{y_n}{L}$$

$$\frac{m\lambda}{d} = \frac{y_n}{L}$$

$$y_n \approx \frac{mL\lambda}{d}$$

$$y_4 - y_3 = \frac{4L\lambda}{d} - \frac{3L\lambda}{d}$$

$$y_4 - y_3 = \frac{L\lambda}{d}$$

$$y_4 - y_3 \approx \boxed{0.600 \text{ mm}}$$

Act III: Proportional Reasoning and Limits of the Model

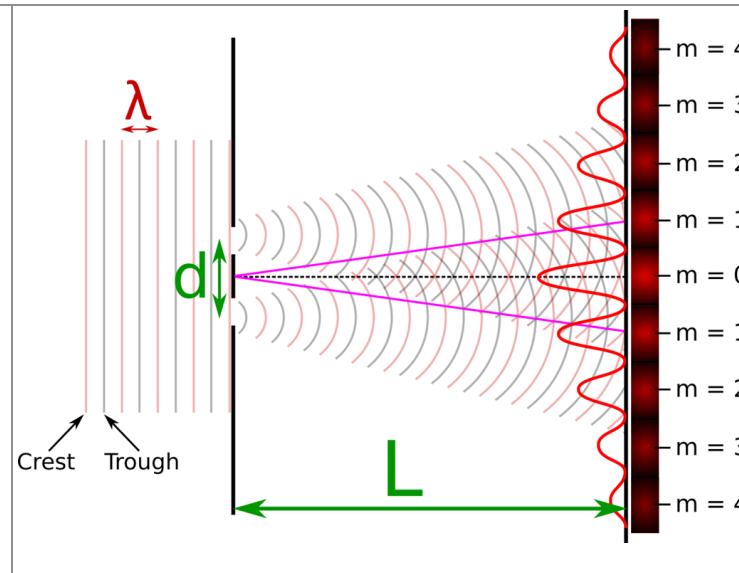
MOVE

WO.L1.2-09:

Problem Statement: Suppose the viewing screen in the figure is moved closer to the double slit. What happens to the interference fringes?

AMPLITUDE / INTENSITY
DECREASES w/ DISTANCE
INTERFERENCE
CYLINDRICAL WAVES

- (1) They fade out and disappear.
- (2) They get out of focus.
- (3) They get brighter and closer together.
- (4) They get brighter and farther apart.
- (5) They get brighter but otherwise do not change.



WO.L1.2-10:

$$y = L \tan \theta_m$$

Problem statement: When observing a double slit experiment, you notice that y_m for some fringe is getting smaller. Which of the following could be causing this?

- (1) The slit width is decreasing.
- (2) The slit width is increasing.
- (3) The wavelength of light is increasing.
- (4) The wavelength of light is decreasing.
- (5) The distance between the slits and screen is increasing.

Then $\tan \theta_m \downarrow$

And $\theta_m \downarrow$

const

$$d \sin \theta_m = m \lambda$$

const

$$d \propto \frac{1}{\sin \theta_m}$$

If $\theta_m \downarrow$

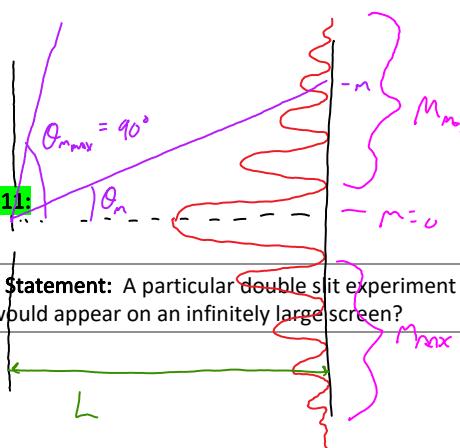
Then $\sin \theta_m \uparrow$

And $\lambda \downarrow$

Then $\sin \theta_m \downarrow$

And $d \uparrow$

WO.L1.2-11:



$$d \sin \theta_m = m \lambda$$

$$d \sin(90^\circ) = M_{\max} \lambda$$

Problem Statement: A particular double slit experiment using 600 nm wavelength light and 50 μm slit separation. How many total fringes would appear on an infinitely large screen?

$$M_{\max} = \frac{d}{\lambda}$$

$$M_{\max} = 83.333$$

$$\text{TOTAL #} = 2M_{\max} + 1$$

$$= \boxed{167 \text{ BRIGHT FRINGES}}$$

BUT M 'S ARE INTEgers

$$\text{So } M_{\max} = 83$$

0

WO.L1.2-12:

$$\text{MAX } \theta = 90^\circ$$

Problem Statement: A particular double slit experiment uses a slit separation d . Is there a limit to the wavelength of the light you can use?

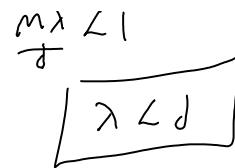
$$M=1 \text{ MUST HAVE } \theta \leq 90^\circ$$

- (1) Yes, $\lambda < d$
- (2) Yes, $\lambda > d$
- (3) No, any slit separation will produce an interference pattern. $\sin \theta_m = m \lambda$

$$\sin \theta_m = \frac{m \lambda}{d}$$

can't
be > 1

So



O

WO.L1.2-13:

$$d\theta = m\lambda$$

$$y_m = L d\theta$$

Problem Statement: In a particular double slit experiment you notice that the small angle approximation is a good approximation. If the wavelength decreases by a factor of 2, what happens to y_2 ?

- (1) It quadruples.
- (2) It doubles.
- (3) It doesn't change.
- (4) It decreases by a factor of 2.
- (5) It decreases by a factor of 4.

$$\frac{y_m}{L} = \frac{m\lambda}{d}$$

If $L = \text{const}$

$$m = \text{const}$$

$$y_m = \frac{m\lambda L}{d}$$

$d = \text{const}$

$$y_m \propto \lambda$$

Conceptual questions for discussion

1. Coming soon to a lecture template near you.

Hints

WO.L1.2-01: No hints.

WO.L1.2-02: No hints.

WO.L1.2-03: No hints.

WO.L1.2-04: No hints.

WO.L1.2-05: No hints.

WO.L1.2-06: No hints.

WO.L1.2-07: No hints.

WO.L1.2-08: No hints.

WO.L1.2-09: No hints.

WO.L1.2-10: No hints.

WO.L1.2-11: No hints.

WO.L1.2-12: No hints.

WO.L1.2-12: No hints.