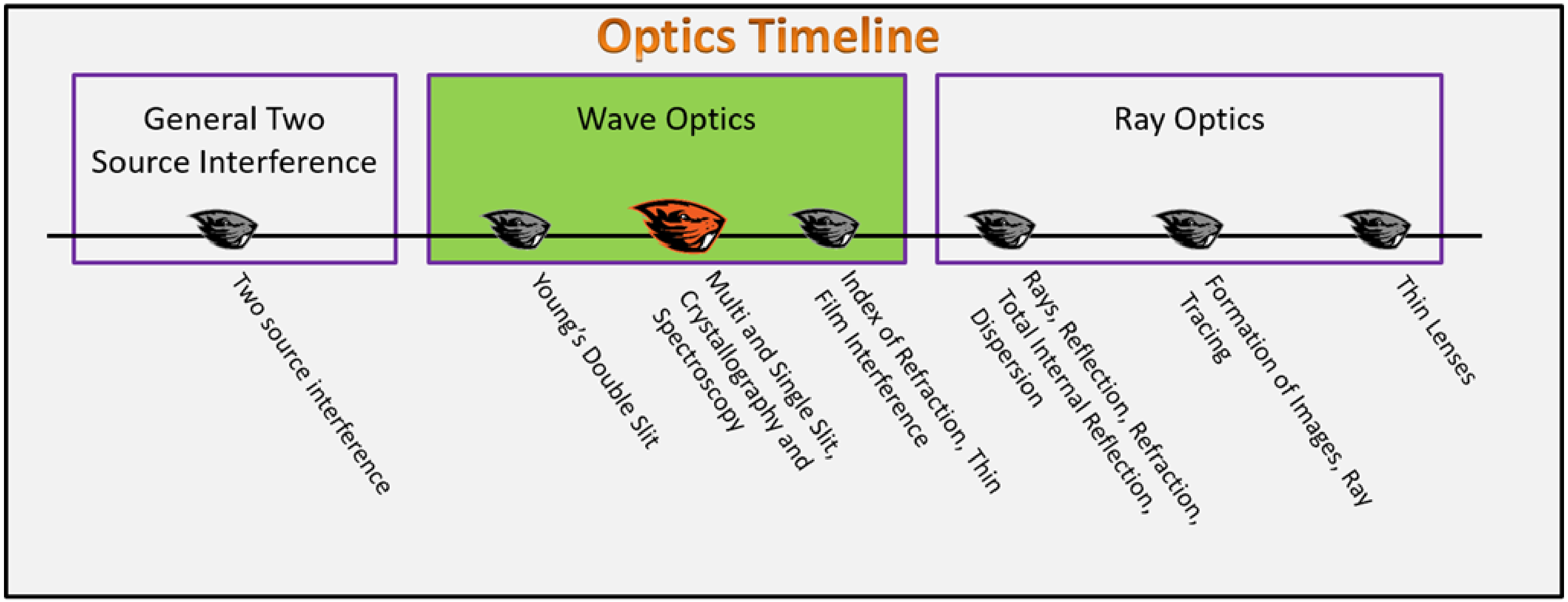
(WO.L2.2-sols) Foundation Stage Solutions

Monday, January 22, 2018 5:44 PM

Wave Optics
Foundation Stage (WO.L2.2)

Lecture 2
Multi and Single Slit, Crystallography and Spectroscopy



Textbook Chapters (* Calculus version)

- **BoxSand** :: KC videos (Single and Multi Slit Interference)
- **Knight** (College Physics : A strategic approach 3rd) :: 17.3 ; 17.5
- ***Knight** (Physics for Scientists and Engineers 4th) :: 33.3 ; 33.4
- **Giancoli** (Physics Principles with Applications 7th) :: 24-1 ; 24-5 ; 24-6 ; 24-7

Warm up

WO.2.L2-1:

Description: Conceptual question about diffraction.

Learning Objectives: [?] - Can you identify the objectives from the previous lecture, and this lecture, that this question is relevant to?

Problem Statement: Which of the following apparatus causes light to diffract?

(1) Double slit.
(2) Multi-slit.
(3) Single slit.
(4) Circular aperture.
(5) Edge of a knife.

Selected Learning Objectives

1. Coming soon to a lecture template near you.

Key Terms

- Multi-slit (i.e. diffraction grating)
- Reflection grating
- Single slit
- Crystallography
- Spectroscopy

Key Equations

Key Concepts

- Coming soon to a lecture template near you.

Questions

Act I: Multiple Slit Interference

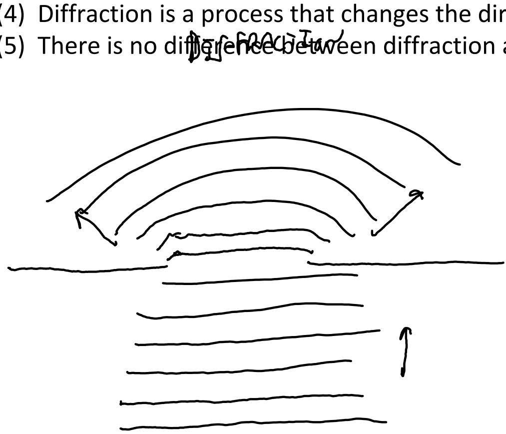
WO.2.L2-2:

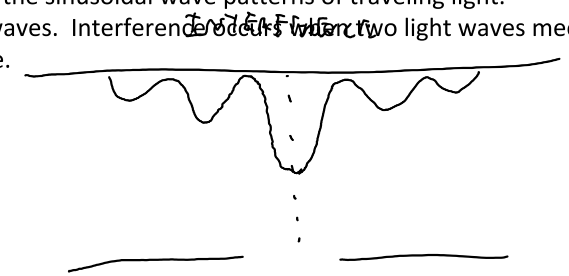
Description: Conceptual question about diffraction and interference. (2 minutes)

Learning Objectives: [?]

Problem Statement: What is the difference between diffraction and interference?

(1) Diffraction refers to multi-slit scattering while interference refers to double slit scattering.
(2) Diffraction is relevant only to single slit apparatus. Interference is relevant to all wave phenomena.
(3) Diffraction is the focusing of light to a spot. Interference is the sinusoidal wave patterns of traveling light.
(4) Diffraction is a process that changes the direction of light waves. Interference occurs when two light waves meet at a point.
(5) There is no difference between diffraction and interference.





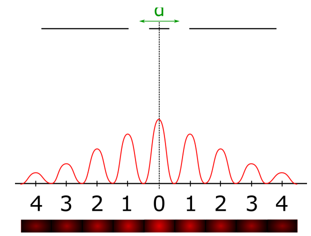
WO.2.L2-3:

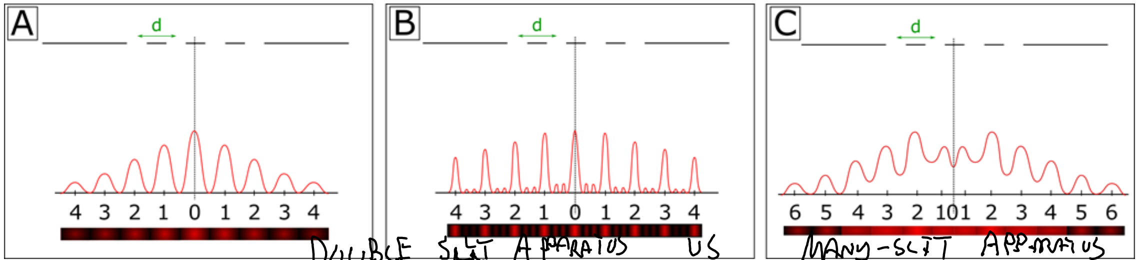
Description: Differentiate between double slit interference patterns and multi-slit interference patterns. (2 minutes + 2 minutes)

Learning Objectives: [?]

Problem Statement: A double slit apparatus produces the interference pattern shown below.

(a) If a second double slit is placed next to the original pair of slits, what does the interference pattern now look like?





DOUBLE SLIT APPARATUS VS MANY-SLIT APPARATUS

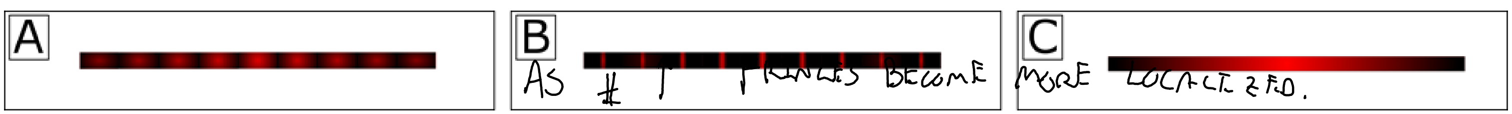
DIFFRACTION GRATINGS

SAME MATH MODLE

$$d \sin\theta_m = m\lambda$$

FOR BRIGHT FRINGES

(b) What does the interference pattern look like if 42 sets of double slits are placed next to each other?



AS # ↑ FRINGES BECOME MORE LOCALIZED.

WO.2.L2-4:

Description: Differentiate between double slit interference patterns and multi-slit interference patterns. (2 minutes)

Learning Objectives: [?]

Problem Statement: While working in a lab, you are tasked to find the wavelength of a laser. Which of the following apparatus would you use to find the wavelength if you wish to have as little uncertainty as possible?

(1) Double slit.
(2) Multi slit.
(3) Single slit.
(4) Ruler.

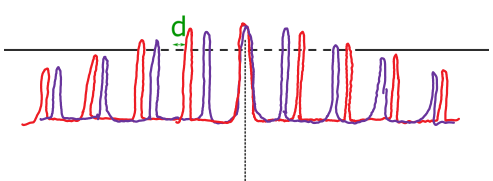
WO.2.L2-5:

Description: Conceptual question about diffraction and white light. Proportional reasoning with diffraction grating math model for PLD. (2 minutes + 4 minutes)

Learning Objectives: [?]

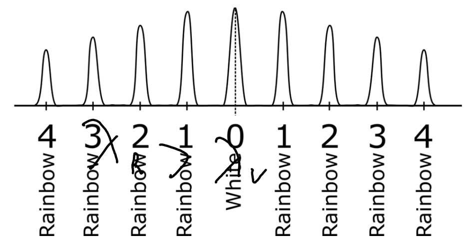
Problem Statement: White light is sent through a diffraction grating and a rainbow is observed as seen below.

(a) What best explains this observation?



(1) Since white light is made up of all wavelengths, the wavelengths interact with each other to produce the rainbow.

(2) Since white light is made up of all wavelengths, the diffraction through the slits separates each wavelength.
(3) Since white light is made up of all wavelengths, the light looks like a rainbow when it shines on any surface.



(b) For each rainbow that is separated from the center bright spot, which of the following is true?

(1) The red side is farthest from the center of the screen, the violet side is closest to the center.
(2) The red side is closest to the center of the screen, the violet side is farthest from the center.
(3) The red side is on the left of the center, the violet side is on the right.
(4) The ride side is on the right of the center, the violet side is on the left.

WO.2.L2-6:

Description: Diffraction grating question given wavelength, lines per distance, screen distance, and fringe of interest, find the angle. (2 minutes + 3 minutes + 3 minutes + 6 minutes)

Learning Objectives: [?]

Problem Statement: 600-nm-light passes through a diffraction grating with **N** = 2500 lines per centimeter. The screen is **L** = 115 cm away from the grating. We eventually wish to determine at what angle the 3rd order bright fringe is located at.

(a) What mathematical model would work for determining the angle of the bright fringes?

(1) d sin(θ) = (m + 1/2) λ
(2) d sin(θ) = m λ
(3) N sin(θ) = (m + 1/2) λ
(4) N sin(θ) = m λ

(b) What is the distance between each slit?

(1) 0.25 m
(2) 0.25 cm
(3) 0.4 mm
(4) 4.0 μm

$d \sin\theta_m = m\lambda$

(c) 600-nm-light passes through a diffraction grating with N = 2500 lines per centimeter. The screen is L = 115 cm away from the grating. **At what angle is the 3rd order bright fringe located at?**

$(4\times10^{-6}\,m)\sin(\theta_3) = 3(600\times10^{-9}\,m)$

(1) 41.1°
(2) 12.4°
(3) 8.63°
(4) 26.7°
(5) 17.3°

$\theta_3 \simeq 26.7°$

w/ $\theta_m = 90°$, 1

$d \sin\theta_m = m\lambda$

(d) How many total fringes are on the screen if the screen is large enough?

$d = m_{max}\lambda$

$m_{max} = \frac{d}{\lambda} = \frac{(4\times10^{-6}\,m)}{(600\times10^{-9}\,m)} = 6.67$

so $m_{max} = 6$

TOTAL # BRIGHT FRINGE = $2m_{max} + 1$

= 13

WO.2.L2-7:

Description: Conceptual question about shining a laser on a DVD and observing an interference pattern. (3 minutes)

Learning Objectives: [?]

Problem Statement: A laser is shone onto the surface of a DVD. The image below shows a picture of the interference pattern produced. Which of the following are related to this system?

(1) PLD.
(2) Double slit.
(3) Diffraction grating.
(4) Single slit.
(5) Reflection grating.

Act II: Single Slit Interference

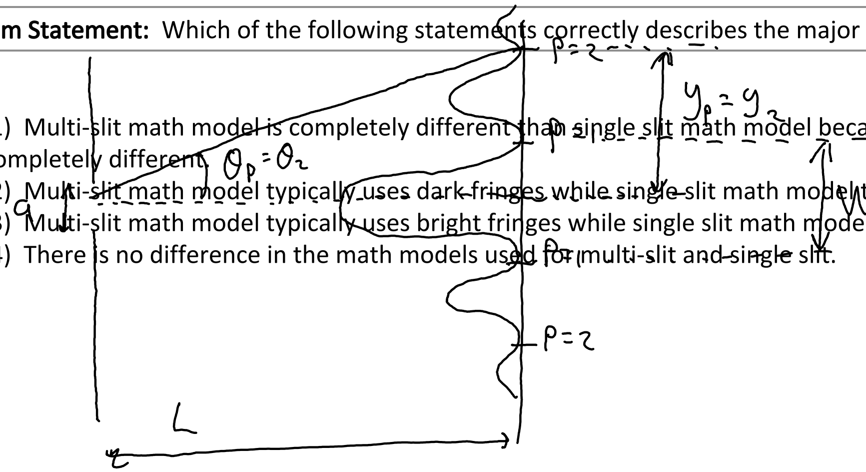
WO.2.L2-8:

Description: Conceptual question comparing multi-slit and single slit analysis. (3 minutes)

Learning Objectives: [?]

Problem Statement: Which of the following statements correctly describes the major difference between single and multi-slit analysis?

(1) Multi-slit math model is completely different than single slit math model because the mechanisms for interference are completely different.
(2) Multi-slit math model typically uses dark fringes while single-slit math model typically uses bright fringes.
(3) Multi-slit math model typically uses bright fringes while single slit math model typically uses dark fringes.
(4) There is no difference in the math models used for multi-slit and single slit.



WO.2.L2-9:

Description: Proportional reasoning with single slit PLD math model. (4 minutes)

Learning Objectives: [?]

$a \sin\theta_p = p\lambda$ $L \tan\theta_p = y_p$ $W = 2y_1$

Problem Statement: A single frequency laser is shone on a single slit of width **a**. If the slit width **a** decreases, what happens to the angle of the 4th dark fringe?

w/ λ & p const — If a ↓ → If θ ↑ (w/ L const) → If y_p ↑ then W ↑

(1) Increases.
(2) Decreases.
(3) Stays the same.

THEN $\theta \uparrow$ THEN $y_p \uparrow$

WO.2.L2-10:

Description: Conceptual question about small angle approximation with single slit apparatus. (3 minutes)

Learning Objectives: [?]

$a \sin\theta_p = p\lambda$

Problem Statement: If the small angle approximation is valid, which of the following relationships are true?

$\sin\theta_p = \frac{p\lambda}{a}$

(1) **a** << **λ**
(2) **a** >> **λ**
(3) **a** ≈ **λ**

MUST BE << 1 FOR SMALL θ } THUS $\lambda << a$

WO.2.L2-11:

Description: Proportional reasoning question for single slit apparatus. (5 minutes)

Learning Objectives: [?]

Problem Statement: Green light is incident on a very thin slit and illuminates a distant screen. Which of the following statements are true if small angles are assumed?

$$\sin\theta_p \approx \theta_p \qquad \tan\theta_p \approx \theta_p$$

(1) If the slit width is doubled, than the width of the central maximum will increase by a factor of two.
(2) If the slit width is doubled, than the width of the central maximum will decrease by a factor of two.
(3) If the distance to the screen is doubled, than the width of the central maximum will increase by a factor of four.
(4) If the distance to the screen is doubled, than the width of the central maximum will decrease by a factor of four.

$$a\sin\theta_p = p\lambda \qquad L\tan\theta_p = y_p \qquad W = 2y_1$$

$$a\,\theta_p \approx p\lambda \qquad L\,\theta_p \approx y_p$$

$$\theta_p \approx \frac{p\lambda}{a}$$

$$\frac{Lp\lambda}{a} \approx y_p$$

$$\frac{L(1)\lambda}{a} \approx y_1$$

$$W \approx \frac{2L\lambda}{a}$$

IF $a \rightarrow 2a$: $w \rightarrow \frac{1}{2}w$

IF $L \rightarrow 2L$: $W \rightarrow 2W$

a

WO.2.L2-12:

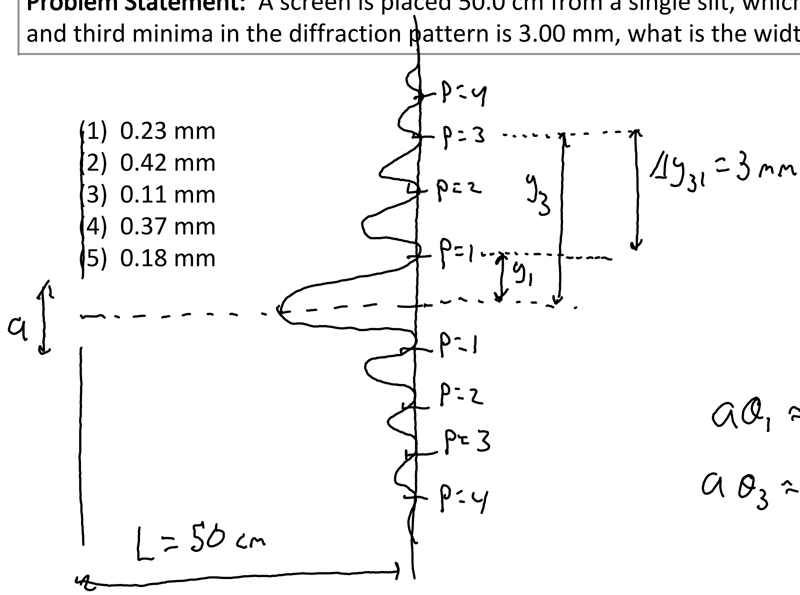
Description: Given screen distance, wavelength, distance between two dark fringes, find slit width for single slit apparatus. (6 minutes)

Learning Objectives: [?]

ASSUME SMALL θ

Problem Statement: A screen is placed 50.0 cm from a single slit, which is illuminated with 690 nm light. If the distance between the first and third minima in the diffraction pattern is 3.00 mm, what is the width of the slit?

(1) 0.23 mm
(2) 0.42 mm
(3) 0.11 mm
(4) 0.37 mm
(5) 0.18 mm



$a\,\theta_p \approx p\lambda \qquad L\,\theta_p \approx y_p$

$\Delta y_{31} \approx L(\theta_3 - \theta_1)$

$\theta_3 - \theta_1 \approx \frac{\Delta y_{31}}{L}$

$a\theta_1 \approx 1\lambda$
$a\theta_3 \approx 3\lambda$ } SUBTRACT $\rightarrow$ $a(\theta_3 - \theta_1) \approx 2\lambda$

$\frac{a\,\Delta y_{31}}{L} \approx 2\lambda$

$a \approx \frac{2L\lambda}{\Delta y_{31}} \approx 2.3 \times 10^{-4}$ m

WO.2.L2-13:

Description: Conceptual question about diffraction comparing light and sound. (4 minutes)

Learning Objectives: [?]

Problem Statement: Which of the following statements best explains why the diffraction of sound is more apparent than the diffraction of light under most circumstances?

(1) Sound requires a physical medium for propagation.
(2) Sound waves are longitudinal, and light waves are transverse.
(3) Light waves can be represented by rays while sound waves cannot.
(4) The speed of sound in air is six orders of magnitude smaller than that of light.
(5) The wavelengths of visible light is considerably smaller than the wavelengths of sound.

$a \sin\theta_p = p\lambda$

w/ a const

IF $\lambda \uparrow$

THEN $\theta_p \uparrow$

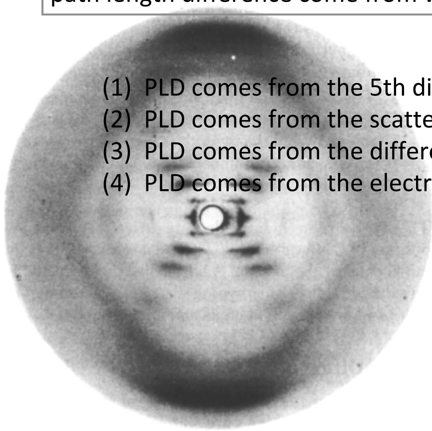
Act III: Applications | Crystallography and Spectroscopy

WO.2.L2-14:

Description: Conceptual question about crystallography. (3 minutes)

Learning Objectives: [?]

Problem Statement: When light shines on the surface of certain types of matter an interference pattern is observed. Where does the path length difference come from when light shines on to these types of matter?



(1) PLD comes from the 5th dimension.
(2) PLD comes from the scattering off different atomic layers of the matter.
(3) PLD comes from the different color of the object compared to the color of the light being shone on it.
(4) PLD comes from the electric and magnetic fields.

WO.2.L2-15:

Description: Conceptual question about crystallography. (4 minutes)

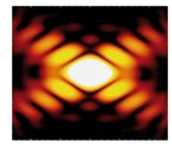
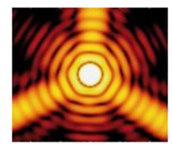
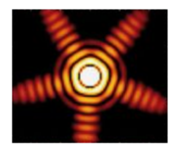
Learning Objectives: [?]

Problem Statement: The scattering pattern for 3 different geometries is shown in the figure. The three geometries, which were used as targets to scatter light off of, are also shown. Match each target with their associated scattering pattern.

B

Scattering Targets		
Icosahedra **A**	Decahedra **B**	Trunc. Octahedra **C**
		

* SYMMETRIES OF INTERFERENCE PATTERN MATCHES SYMMETRIES OF OBJECT

Interference Patterns	Scattering Targets
	A
	
	

WO.2.L2-16:

~ SHARPEST FRINGES

Description: Conceptual question about spectroscopy. (3 minutes)

Learning Objectives: [?]

Problem Statement: Benny the Beaver is working at Castor National Laboratories studying a star in the Gemini constellation. Which device should Benny use to separate the light from the star into its constituent wavelengths?

(1) Double slit apparatus.
(2) Diffraction grating apparatus.
(3) Single slit apparatus.
(4) Reflection grating apparatus.
(5) A katana.

WO.2.L2-17:

~ ALL OBJECTS w/ T > 0K RADIATE EM WAVES
~ DOPPLER SHIFT
~ P & T RELATED VIA EQUATIONS OF STATE
"FINGER PRINT" OF ELEMENTS

Description: Conceptual question about spectroscopy. (3 minutes)

Learning Objectives: [?]

Problem Statement: Physicist analyze the electromagnetic spectrum of astrophysical objects to make inferences about which of the following?

(1) Temperature.

(2) Velocity.
(3) Gas pressure.
(4) Overall composition.

WO.2.12-18:

Description: Conceptual question about spectroscopy. (3 minutes)

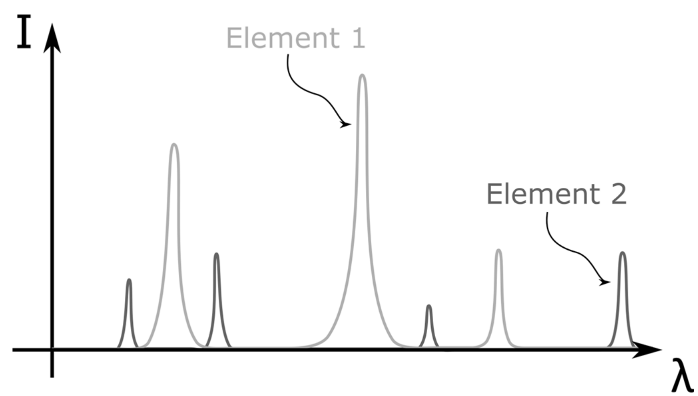
Learning Objectives: [?]

$I_1 > I_2$

Problem Statement: The spectral lines of a distant star are shown to match only two elements. What features of the lines can be used to determine the percentage of each element in the star?

SO PROBABLY MORE OF ELEMENT 1 PRESENT

(1) Frequency.
(2) Wavelength.
(3) Intensity.
(4) Doppler shift.



WO.2.L2-19:

$C = f\lambda$

Description: Conceptual question about spectroscopy. (3 minutes)

$C = 3\times10^8$ m/s A CONSTANT

Learning Objectives: [?]

SO $\lambda \propto \frac{1}{f}$

Problem Statement: What feature of the spectral lines could be used to determine the relative motion of the star to Earth?

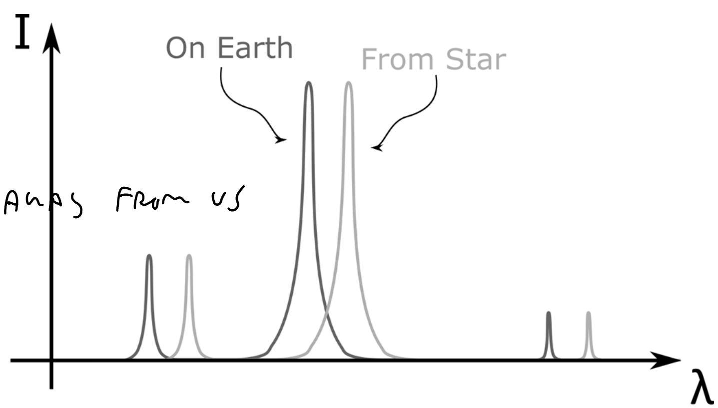
IF $\lambda \uparrow$ THEN $f \downarrow$

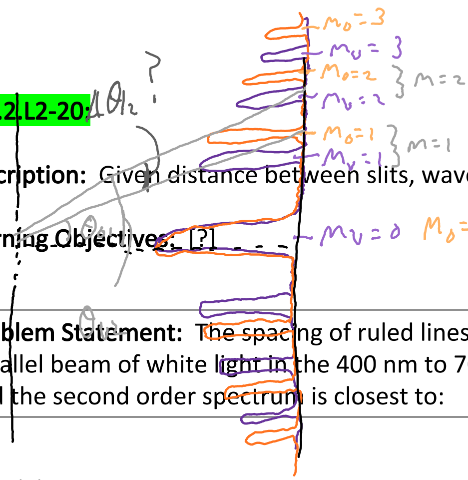
(1) Frequency.
(2) Wavelength.
(3) Intensity.
(4) Doppler shift.

SO IF $f \downarrow$

THEN $|\Delta \vec{f}| \uparrow$

MOVING AWAY FROM US





$d \sin\theta_n = m\lambda$

V: $d \sin\theta_{v2} = 2\lambda_v$ → $\theta_{v2} \approx 24.901°$

O: $d \sin\theta_{o1} = 1\lambda_o$ → $\theta_{o1} \approx 21.618°$

$\Delta\theta = 3.3°$

WO.2.L2-20:

Description: Given distance between slits, wavelengths, and fringes of interest, find angular separation. (8 minutes)

Learning Objectives: [?]

Problem Statement: The spacing of ruled lines on a diffraction grating is 1900 nm. The grating is illuminated at a normal incidence with a parallel beam of white light in the 400 nm to 700 nm wavelength band. The angular width of the gap between the first order spectrum and the second order spectrum is closest to:

(1) 3.3°
(2) 4.3°
(3) 5.3°
(4) 6.3°
(5) 2.3°

Conceptual questions for discussion

1. Coming soon to a lecture template near you.

Hints

WO.2.L2-1: No hints.

WO.2.L2-2: No hints.

WO.2.L2-3: No hints.

WO.2.L2-4: No hints.

WO.2.L2-5: No hints.

WO.2.L2-6: No hints.

WO.2.L2-7: No hints.

WO.2.L2-8: No hints.

WO.2.L2-9: No hints.

WO.2.L2-10: No hints.

WO.2.L2-11: No hints.

WO.2.L2-12: No hints.

WO.2.L2-13: No hints.

WO.2.L2-14: No hints.

WO.2.L2-15: No hints.

WO.2.L2-16: No hints.

WO.2.L2-17: No hints.

WO.2.L2-18: No hints.

WO.2.L2-19: No hints.

WO.2.L2-20: No hints.